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PROBABILISTIC MODEL CODE

Part 1 - BASIS OF DESIGN

Contents

1. Introduction

This part treats the general principles for a probabilistic design of load bearing structures. The more detailed aspects dealing with the probabilistic description of loads are treated in part 2. In the same way the probabilistic description of structural resistance parameters is treated in part 3.
This part doesn't give detailed information about methods for the calculation of probabilities. It is assumed that the user of a probabilistic code is familiar with such methods. A clause on the interpretation of probabilities treated in this document is provided in Annex D.

2. Requirements

2.1. Basic requirements

Structures and structural elements shall be designed, constructed and maintained in such a way that they are suited for their use during the design working life and in an economic way.

In particular they shall, with appropriate levels of reliability, fulfil the following requirements:

- They shall remain fit for the use for which they are required (*serviceability limit state requirement*)
- They shall withstand extreme and/or frequently repeated actions occurring during their construction and anticipated use (*ultimate limit state requirement*)
- They shall not be damaged by accidental events like fire, explosions, impact or consequences of human errors, to an extent disproportionate to the triggering event (*robustness requirement*, see Annex A).

2.2. Reliability differentiation

The expression "*with appropriate levels of reliability*" used above means that the degree of reliability should be adopted to suit the use of the structure, the type of structure or structural element and the situation considered in the design, etc.

The choice of the various levels of reliability should take into account the possible consequences of failure in terms of risk to life or injury, the potential economic losses and the degree of social inconvenience, as described in chapter 8. It should also take into account the amount of expense and effort required to reduce the risk of failure. It is further noted, that the term "failure" as used in this document refers to either inadequate strength or inadequate serviceability of the structure.

The consequences of a failure generally depend on the mode of failure, specially in those cases when the risk to human life or injury exists.

In order to provide a structure corresponding to the requirements and to the assumptions made in the design, appropriate *quality measures* shall be adopted. These measures comprise definition of reliability requirements, organisational measures and controls at the stages of design, execution and use and the maintenance of the structure.

2.3. Requirements for durability

The durability of the structure in its environment shall be such that it remains fit for use during its design working life. This requirement can be considered in one of the following ways:

a) By using materials that, if well maintained, will not degenerate during the design working life.
b) By giving such dimensions that deterioration during the design working life is compensated.
c) By chosing a shorter lifetime for structural elements, which may be replaced one or more times during the design working life.
d) By inspection at fixed or condition dependent intervals and appropriate maintenance activities.

In all cases the reliability requirements for long and short term periods should be met. Analysis aspects on durability are described in Annex B.

3. Principles of limit state design

3.1. Limit states and adverse states

The structural performance of a whole structure or part of it should be described with reference to a specified set of limit states which separate desired states of the structure from adverse states.

The limit states are divided into the following two basic categories:

- the *ultimate limit states*, which concern the maximum load carrying capacity as well as the maximum deformability
- the *serviceability limit states*, which concern the normal use.

The exceedance of a limit state may be irreversible or reversible. In the irreversible case the damage or malfunction associated with the limit state being exceeded will remain until the structure has been repaired. In the reversible case the damage or malfunction will remain only as long as the cause of the limit state being exceeded is present. As soon as this cause ceases to act, a transition from the adverse state back to the desired state occurs.

It is further noted here that in some cases a limit between the aforementioned limit state types may be defined This can be done by an artificial discretization of a the continuous situation between the serviceability and the ultimate limit state. By applying such a procedure a so-called *partial damage limit state*" can be defined. For example in case of earthquake damage of plant structures such limit state is associated to the safe shut down of the plant.

Ultimate limit states may correspond to the following adverse states:

- loss of equilibrium of the structure or of a part of the structure, considered as a rigid body (eg. overturning)
- attainment of the maximum resistance capacity of sections, members or connections by rupture or excessive deformations
- rupture of members or connections caused by fatigue or other time-dependent effects instability of the structure or part of it
- sudden change of the assumed structural system to a new system, (eg. snap through)

The exceedance of an ultimate limit state is almost always irreversible and the first time that this occurs causes failure.

Serviceability limit states may correspond to the following adverse states:

- local damage (including cracking) which may reduce the durability of the structure or affect the efficiency or appearance of structural or non-structural elements.
- observable damage caused by fatigue or other time dependent effects
- unacceptable deformations which affect the efficient use or appearance of structural or non-structural elements or the functioning of equipment.
  excessive vibrations which cause discomfort to people or affect non-structural
- elements or the functioning of equipment

In the cases of permanent local damage or permanent unacceptable deformations the exceedance of a serviceability limit state is irreversible and the first time that this occurs causes failure.

In other cases the exceedance of a serviceability limit state may be reversible and then failure occurs:

a) the first time the serviceability limit state is exceeded, if no exceedance is considered as acceptable
b) if exceedance is acceptable but the time when the structure is in the undesired state is longer than specified
c) if exceedance is acceptable but the number of times that the serveciability limit state is exceeded is larger than specified
d) if a combination of the above criteria occur.

These cases may involve temporary local damage (eg. temporarily wide cracks), temporary large deformations and vibrations. Limit values for the serviceability limit state should be defined on the basis of *utility considerations*.

3.2. Limit State Function

For each specific limit state the relevant basic variables should be identified, i.e. the variables which characterize:

- actions and environmental influences
- properties of materials and soils
- geometrical parameters

Such variables may be time dependent. Models, which describe the behaviour of a structure, should be established for each limit state. These models include mechanical models, which describe the structural behaviour, as well as other physical or chemical models, which describe the effects of environmental influences on the material properties. The parameters of such models should in principle be treated in the same way as basic variables.

Serviceability constaints (limit values according to 4.1) should in principle be regarded as random and may in many cases be treated in the same way as basic variables.

Where calculation models are available, the limit state can be described with aid of a function, g, of the basic variables $\underline{X}(t) = X_1(t), X_2(t), \ldots$ so that

$$g\,(\underline{X}(t)) = 0 \qquad (1)$$

Eq. (1) is called the limit state equation, and

$$g\,(\underline{X}(t)) < 0 \qquad (2)$$

identifies the adverse state.

In a component analysis where there is one dominating failure mode the limit state condition can normally be described by one equation according to eq. (1). In a system analysis, where more than one failure mode may be determining, there are several such equations.

3.3. Design situations

Actions, environmental influences and structural properties may vary with time. Such variations, which occur throughout the lifetime of the structure, should be considered by selected design situations, each one representing a certain time intervall with associated hazards, conditions and relevant structural limit states.

The design situations may be classified as:

Persistent situations, which refer to conditions of normal use of the structure and are generally related to the working life of the structure.
Transient situations, which refer to temporary conditions of the structure, in terms of its use or its exposure.
Accidental situations, which refer to exceptional conditions of the structure or its exposure.

4. Basis of uncerainty modelling

4.1. Basic variables

The calculation model for each limit state considered should contain a specified set of basic variables, i.e. physical quantities which characterize actions and environmental influences, material and soil properties and geometrical quantities. The model should also contain model parameters which characterize the model itself and which are treated as basic variables (compare 4.2). Finally there are also parameters which describe the requirements (e.g. serviceability constraints) and which may be treated as basic variables. The basic variables (in the wide sence given above) are assumed to carry the entire input information to the calculation model.

The basic variables may be random variables (indlucing the special case deterministic variables) or stochastic processes or random fields. Each basic variable is defined by a number of parameters such as mean, standard deviation, parameters determining the correlation structure etc.

4.2. Types of uncertainty

Uncertainties from *all essential sources* must be evaluated and integrated in a basic variable model. Types of uncertainty to be taken into account are:

- intrinsic physical or mechanical uncertainty
- statistical uncertainty, when the design decisions are based on a small sample of observations or when there are other similar conditions
- model uncertainties (see 5.6).

Within given classes of structural design problems the types of probability distributions of the basic variables should be standardized. These standardizations are defined in the parts 2 and 3 of the probabilistic model code.

4.3. Definition of populations

The random quantities within a reliability analysis should always be related to a meaningfull and consistent set of populations. The description of the random quantities should correspond to this set and the resulting failure probability is only valid for the same set.

The basis for the definition of a population is in most cases the physical background of the variable. Factors which may define the population are:

- the nature and origin of a random quantity
- the spatial conditions (e.g. the geographical region considered)
- the temporal conditions (e.g. the intended time of use of the structure considered)

The choice of a population is to some extent a free choice of the designer. It may depend on the objective of the analysis, the amount and nature of the available data and the amount of work that can be afforded.

In connection with theoretical treatment of data and with the evaluation of observations it is often convenient to divide the largest population into sub-populations which in turn are further divided in smaller sub-populations etc. Then it is possible to study and distinguish variability within a population and variability between different populations.

In an analysis for a specific structure it may be efficient to define a population as small as possible as far as use, shape and location of the structure are concerned (microzonation). When the results are used for design in a national or international code, it may be necessary or convenient to put the sub-populations together to the large population again in order not to get too complicated rules (randomizing). This means that the variability within the population is increased.

4.4. Hierarchy of uncertainty models

This section contains a convenient and recommended mathematical description in general terms of a hierarchical model which can be used for different kinds of actions and materials. The details of this model have to be stated more precisely for each specific variable. The model is associated with a hierarchical set of subpopulations.

The hierarchical model assumes that a random quantity X can be written as a function of several variables, each one representing a specific type of variability:

$$X_{ijk} = f(Y_i, Y_{ij}, Y_{ijk}) \qquad (3)$$

The Y represent various origins, time scales of fluctuation or spatial scales of fluctuation.

For instance Y_i may represent the building to building variation, Y_{ij} the floor to floor variation in building i and Y_{ijk} the point to point variation on floor j in building i.

In a similar way, Y_i may represent the constant in time variability, Y_{ij} a slowly fluctuating time process and Y_{ijk} a fast fluctuating time process.

5. Models for physical behaviour

5.1. General

Calculation models shall describe the structure and its behaviour up to the limit state under consideration, accounting for relevant actions and environmental influences. Models should generally be regarded as simplifications which take account of decisive factors and neglect the less important ones.

One can often distinguish between:

- action models
- structural models which give action effects (internal forces, moments etc.)
- resistance models which give resistances corresponding to the action effects, and are based on.
- material models and geometry models .

However, in some cases it is not possible or convenient to make this distinction, for example, if the instability or loss of equilibrium of an entire structural system is studied or if interactions between loads and structural response are of interest.

5.2. Action models

A complete action model should describe several properties of the action such as its magnitude, position, direction, duration etc. In some cases there is an interaction between the different properties and also between these properties and the response of the structure. Such interactions should be taken into account.

The magnitude F of an action may often be described by two different types of variables so that

$$F = \varphi (F_o, W) \qquad (4)$$

where φ is an appropriate function and

F_o is a basic action variable, often with time and space dependent variations (random or non-random) and is generally independent of the structure

W is a random or non-random variable or a random field which may depend on the structural properties and which transformes F_o to F.

Eq. (4) should be regarded as a symbolic expression where F_o and W may represent several variables.

One example may be snow load where F_o is the time dependent snow load on ground and W is the conversion factor for snow load on ground to snow load on roof which normally is assumed to to be time independent.

Further information on action models is provided in part 2. It is noted that action models may include material properties (earthquake action depends for example on material damping).

5.3. Geometrical models

A structure can generally be described by a model consisting of one-dimensional elements (beams, columns, cables, arches, etc), two-dimensional elements (slabs, walls, shells, etc) and three-dimensional elements.

The geometrical quantities which are included in the model generally refer to nominal values, i.e. the values given in drawings, descriptions etc. Normally, the geometrical quantities of a real structure differ from their nominal values, i.e. the structure has geometrical imperfections. If the structural behaviour is sensitive to such imperfections, these shall be inlcuded in the model.

In many cases the deformation of a structure causes significant deviations from nominal values of geometrical quantities. If such deformations are of importance for the structural behaviour, they have to be considered in the design in principally the same way as imperfections. The effects of such deformations are generally denoted *geometrically nonlinear* or *second order effects* and should be accounted for.

5.4. Material models

When strength or stiffness is considered the material model normally consists of relations between forces or stresses and deformations i.e *costitutive relationships*. The parameters of such relations are modulus of elasticity, yield limit, ultimate strength etc. which generally are considered as random variables, Sometimes they are time dependent or space dependent. There is often an correlation between the parameters e.g. the modulus of elasticity and the ultimate strength of concrete.

Other material properties, e.g. resistance against material deterioration may often be treated in a similar way. However the principles are strongly dependent on type of material and the property considered.

Further information related to models of several material types is given in part 3.

5.5. Mechanical models

The following mechanical models may be classified

a) models describing static response
b) models decribing dynamic response
c) models for fatigue

a) models describing static response

In almost all design calculations some assumptions concerning the relation between forces or moments and deformations (or deformation rates) are necessary. These assumptions can vary and depend on the purpose and type of calculation. The most general relationship regarding structural response is considered to be elastic) developing into plastic behaviour in certain parts of the structure at high action effects. In other parts of the structure intermediate stages occur. Such relationships may be used generally. However the use of any theory taking into account in-elastic or post-critical behaviour may have to take into account repetitions of variable actions that are free. Such actions may cause great variations of the action effects, repeated yielding and exhaustion of the deformation capacity.

The theory of elasticity may be regarded as a simplification of a more general theory and may generally be used provided that forces and moments are limited to those values, for which the behaviour of the structure is still considered as elastic. However, the theory of elasticity may also be used in other cases if it is applied as a conservative approximation.

Theories in which fully developed plasticity is assumed to occur in certain zones of the structure (plastic hinges in beams, yield lines in slabs, etc) may also be used, provided that the deformations which are needed to ensure plastic behaviour, occur before the ultimate limit state is reached. Thus theory of plasticity should be used with care to determine the load carrying capacity of a structure, if this capacity is limited by:

- brittle failure
- failure due to instability

b) models for dynamic response

In most cases dynamic response of a structure is caused by a rapid variation of the magnitude, position or direction of an action However, a sudden change of the stiffness or resistance of a structural element may also cause dynamic behaviour.

The models for dynamic response consist in general of:

- a stiffness model
- a damping model
- an inertia model

c) *models for fatigue*

Fatigue models are used for the description of fatigue failures caused by fluctuating actions. Two types of models are distinguished:

a) S-N model based on experiments
b) fracture mechanics model

It is further noted here, that other types of degradation such as chemical attack or fire can modify the parameters entering the aforementioned models or the models themselves.

5.6. Model uncertainties

A calculation model is a physically based or empirical relation between relevant variables, which are in general random variables:

$$Y = f(X_1, X_2, \dots X_n) \qquad (5)$$

Y	=	model output
$f(\ )$	=	model function
X_i	=	basic variables

The model f (...) may be complete and exact, so that, if the values of X_i are known in a particular experiment (from measurements), the outcome Y can be predicted without error. This, however, is not normally the situation. In most cases the model will be incomplete and inexact. This may be the result of lack of knowledge, or a deliberate simplification of the model, for the convenience of the designer. The difference between the model prediction and the real outcome of the experiment can be written down as:

$$Y = f'(X_1 \dots X_n, \theta_1 \dots \theta_m) \qquad (6)$$

θ_i are referred to as parameters which contain the model uncertainties and are treated as random variables. Their statistical properties can in most cases be derived from experiments or observations. The mean of these parameters should be determined in such a way that, on average, the calculation model correctly predicts the test results.

6. Reliability

6.1. Reliability measures

A standard reliability measure may be chosen to be the *generalized reliability index*. It is defined as:

$$\beta = - \Phi^{-1} (P_f) \qquad (7)$$

where P_f is the probability of failure

$\Phi^{-1}(\cdot)$ is the inverse Gaussian distribution

Another equivalent reliability measure is the probability of the complement of the adverse event

$$P_s = 1 - P_f \qquad (8)$$

The probability P_f should be calculated on the basis of the standardized joint distribution type of the basic variables and the standardized distributional formalism of dealing with both model uncertainty and statistical uncertainty.

In special situations other than the standardized distribution types can be relevant for the reliability evaluation. In such cases the distributional assumptions must be tested on a suitable representative set of observation data.

Reliability analysis principles including time-dependent reliability problems are described in Annex C.

6.2. Component reliability and system reliability

Component reliability is the reliability of one single structural component which has one dominating failure mode.

System reliability is the reliability of a structural system composed of a number of components or the reliability of a single component which has several failure modes of nearly equal importance. The following type of systems can be classified:

- -*redundant systems* where the components are "fail safe", i.e. local behaviour of one component does not directly result in failure of the structure;
- -*non-redundant* systems where local failure of one component leads rapidly to failure of the structure.

Probabilistic structural design is primarily concerned with *component behaviour*. System behaviour is, however, of concern because it is usually the most serious consequence of structural failure. Therefore the likelihood of system failure following an initial component failure should be assessed. In particular, it is necessary to determine the system characteristics in relation to damage tolerance or robustness with respect to accidental events. The requirements for the reliability of the components of a system should depend upon the system characteristics.

A probabilistic system analysis should therefore be carried out to establish:

- the redundancy (alternate load-carrying paths)
- the state and complexity of the structure (multiple failure modes).

Furher aspects on system reliability are provided in Annex C.

6.3. Methods for reliability analysis and calculation

The numerical value of the reliability measure is obtained by a reliability analysis and calculation method (see Annex C). The reliability method used should be capable of producing a sensitivity analysis including importance factors for uncertain parameters. The choice of the method should be in general justified. The justification can be for example based by another relevant computation method or by reference to appropriate literature.

Due to the computational complexity a method giving an approximation to the exact result is generally applied. Two fundamental accuracy requirements are:

- Overestimation of the reliability due to use of an approximative calculation method shall be within limits generally accepted for the specific type of structure.
- The overestimation of the reliability index should not exceed 5 % with respect to the target level.

The accuracy of the reliability calculation method is linked to the sensitivity with respect to structural dimensions and material properties in the resulting design.

7. Target Reliability

7.1. General Aspects

In terms of a reliability based approach the structural risk acceptance criteria correspond to a required minimum reliability herein defined as *target reliability*. The requirements to the safety of the structure are consequently expressed in terms of *the accepted minimum reliability index* or *the accepted maximum failure probability*.

In a rational analysis the target reliability is considered as a control parameter subject to *optimization*. The parameter assigns a particular investment to the material placed in the structure. The more material - invested in right places - the less is the expected loss. Such optimization is mainly possible when economic loss components dominate over life, injury, and culture components. When the expected loss of life or limb is important, the optimal reliability level becomes more controversional. Frequently, this leads to the problem of the economic equivalent of human life; *risk-benefit analyses* are then applied to circuvent this difficulty; the *reliability of the system is translated into the cost per life saved*. The target reliability may then be chosen such that the cost per life saved is at acceptable levels (for example comparable to other similar systems).

In a practical approach the required reliability of the structure is controlled by:
i) a set of assumptions about *quality assurance* and *quality management* measures; these measures are for example related to design and construction supervision and are intended to avoid *gross errors*.
ii) formal failure probability requirements, *conditional upon these assumptions*, defined by specified target values for the various classes of structures and structural members.

7.2. Recommendations

Target reliability values are provided in the next paragraphs. They are based on optimization procedures and on the assumption that for allmost all engineering facilities the only reasonable reconstruction policy is systematic rebuilding or repair.

7.2.1. Ultimate Limit States

Target reliability values for ultimate limit states are proposed in Table 1. The values in Table 1 are obtained based on cost benefit analysis for the public at characteristic and representative but simple example structures and are compatible with calibration studies and statistical observations.

Table 1: Tentative target reliability indices β (and associated target failure rates) related to one year reference period and ultimate limit states

1	2	3	4
Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
Large (A)	β=3.1 ($p_F \approx 10^{-3}$)	β=3.3 ($p_F \approx 5\ 10^{-4}$)	β=3.7 ($p_F \approx 10^{-4}$)
Normal (B)	β=3.7 ($p_F \approx 10^{-4}$)	β=4.2 ($p_F \approx 10^{-5}$)	β=4.4 ($p_F \approx 5\ 10^{-6}$)
Small (C)	β=4.2 ($p_F \approx 10^{-5}$)	β=4.4 ($p_F \approx 5\ 10^{-6}$)	β=4.7 ($p_F \approx 10^{-6}$)

The shadowed value in Table 1 should be considered as the most common design situation.
In order to make the right choice in this table the following guidelines may be of help:

- **Consequence classes**

A classification into consequenze classes is based on the ratio ρ defined as the ratio between total costs (i.e. construction costs plus direct failure costs) and construction costs.

Class 1 Minor Consequences: ρ is less than approximately 2
Risk to life, given a failure, is small to negligible and economic consequences are small or negligible (e.g. agricultural structures, silos, masts);
Class 2 Moderate Consequences: ρ is between 2 and 5.
Risk to life, given a failure, is medium or economic consequences are considerable (e.g. office buildings, industrial buildings, apartment buildings).
Class 3 Large Consequences: ρ is between 5 and 10.

Risk to life, given a failure, is high, or economic consequences are significant (e.g. main bridges, theaters, hospitals, high rise buildings).

If ρ is larger than 10 and the absolute value of H also is large, the consequences should be regarded as extreme and a full cost benefit analysis is recommended. The conclusion might be that the structure should not be build at all.

One should be aware of the fact that failure consequences also depend on the type of failure, which can be classified according to:

a) ductile failure with reserve strength capacity resulting from strain hardening
b) ductile failure with no reserve capacity
c) brittle failure

Consequently a structural element which would be likely to collapse suddenly without warning should be designed for a higher level of reliability than one for which a collapse is preceded by some kind of warning which enables measures to be taken to avoid severe consequences.
The values given relate to the structural system or in approximation to the dominant failure mode or structural component dominating system failure. Therefore, structures with multiple, equally important failure modes should be designed for a higher level of reliability.

♦ **Relative cost of safety measures classificaton**
The normal class (B) should be associated with:

- medium variabilities of the total loads and resistances ($0.1 < V < 0.3$),
- relative cost of safety measure
- normal design life and normal obsolesce rate composed to construction costs of the order of 3%

The given values are for structures or structural elements as designed (not as built). Failures due to human error or ignorance and failures due to non-structural causes are not covered by table 1.
Values outside the given ranges may lead to a higher or lower classification. In particular attention may be given to the following aspects:

♦ *Degree of Uncertainty*
A large uncertainty in either loading or resistance (coefficients of variation larger then 40 %), as for instance the case of many accidental and seismic situations, a lower reliability class should be used. The point is that for these large uncertainties the additional costs to achieve a high reliability are prohibitive. If on the other hand both acting and resisting variables have coefficients of variation smaller than 10%, like for most dead loads and well-known small resistance variability, a higher class can be achieved by very little effort and this should be done.
♦ *Quality assurance* and *inspections*

Quality assurance (for new structures) and inspections (for existing structures) have an increasing effect on costs. This will lead to a lower reliability class. On the other hand, due to QA and inspections the uncertainty will normally decrease and a higher class becomes economically more attractive. General rules are difficult to give.

- *Existing structures*

For existing structures the costs of achieving a higher reliability level are usually high compared to structures under design. For this reason the target level for existing structures usually should be lower.

- *Service life and/or obsolesce*

For structures designed for short service life or otherwise rapid obsolesce (say less than 10 years) the beta-values can be lowered by one or half a class.

By definition serviceability failures are not associated with loss of human life or limb. For existing structures the demand will be more related to the actual situation in performance and use. No general rules are given in this document.

7.2.2. Serviceability Limit State

When setting target values for serviceability limit states (SLS) it is important to distinguish between irreversible and reversible serviceability limit states. Target values for SLS can be derived based on decision analysis methods.

For irreversible serviceability limit states tentative target values are given in Table 2. A variation from the target serviceability indexes of the order of 0.3 can be considered. For reversible serviceability limit states no general values are given.

Table 2: Tentative target reliability indices (and associated probabilities) related to one year reference period and irreversible serviceability limit states

Relative Cost of Safety Measure	Target Index (irreversible SLS)
High	$\beta=1.3 (p_F \approx 10^{-1})$
Normal	$\beta=1.7 (p_F \approx 5\ 10^{-2})$
Low	$\beta=2.3 (p_F \approx 10^{-2})$

8. Annex A: The Robustness Requirement

8.1. Introduction

In clause 3.1 the following robustment requirement has been formulated:

> *"A structure shall not be damaged by events like fire explosions or consequences of human errors, deterioration effects, etc. to an extend disproportionate to the severeness of the triggering event".*

This annex is intended to give some further guidance. No attention is being paid to terrorist actions and actions of war. The general idea is that, whatever the design, proper destructive actions can always be succesful.

8.2. Structural and nonstructural measures

In order to attain adequate safety in relation with accidental loads one or more of the following strategies may be followed:

1. reduction of the probability that the action occurs or reduction of the action intensity (prevention)
2. reduction of the effect of the action on the structure (protection)
3. making the structure strong enough to withstand the loads
4. limiting the amount of structural damage
5. migitation of the consequences of failure

The strategies 1, 2 and 5 are so called non-structural measures. These measures are considered as being very effective for some specific accidental action.

The strategies 3 and 4 are so called structural measures. In general strategy 3 is extremely expensive in most cases. Strategy 4, on the other hand accepts some members to fail, but requires that the total damage is limited. This means that the structure should have sufficient redundancy and possibilities to mobilise so called alternative load paths.

In the ideal design procedure, the occurrence and effects of an accidental action (impact, explosion, etc.) are simulated for all possible action scenarios. The damage effect of the structural members is calculated and stability of the remaining structure assessed. Next the consequences are estimated in terms of number of casualties and economic losses. Various measures can be compared on the basis of economic criteria.

8.3. Simplified design procedure

The approach sketched in A2 has two disadvantages:

(1) it is extremely complicated
(2) it does not work for unforseenable hazards

As a result other more global design strategies have been developed, like the classical requirements on sufficient ductility and tying of elements.

Another approach is that one considers the situation that a structural element (beam, column) has been damaged, by whatever event, to such an extend that its normal load bearing capacity has vanished almost completely. For the remaining part of the structure it then required that fore some relatively short period of time (repair period T) the structure can withstand the "normal" loads with some prescribed reliability:

$$P(R < S \text{ in } T \mid \text{one element removed}) < p_{target} \quad \text{(A1)}$$

The target reliability in (A1) depends on:

- the normal safety target for the building
- the period under consideration (hours, days or months)
- the probability that the element under consideration is removed (by other causes then already considered in design).

The probability that some element is removed by some cause, not yet considered in design, depends on the sophistication of the design procedure and on the type of structure. For a conventional structure it should, at least in theory, be possible to include all relevant collapse origins in the design. Of course, it will always be possible to think of failure causes not covered by the design, but those will have a remote likelihood and may be disregarded on the basis of decision theoretical arguments. For unconventional structures this certainly will not be the case.

8.4. Recommendation

For *unconventional* structures, as for instance large structures, the probability of having some unspecified failure cause is substantial. If in addition new materials or new design concepts are used, unexpected failure causes become more likely. This would indicate that for unconventional structures the simplified approach should be recommended.

For *conventional* structures there is a choice:

(1) one might argue that, as one never succeeds in dealing with all failure causes explicitly in a satisfactory way, it has no use to make refined analyses including system effect, accidental actions and so on; this leads to the use of the simplified procedure.
(2) one might also eliminate the use of an explicit robustness requirement (A1) as much as possible by taking into the design as many aspects explicitly as possible.

Stated as such it seems that the second approach is more rational, as it offers the possibility to reduce the risks in the most economical way, e.g. by sprinklers (for fire), barriers (for collision), QA (for errors), relief openings (for explosions), artificial damping (for earth quake), maintenance (for deterioration) and so on.

9. Annex B: Durability

9.1. Probabilistic Formulations

Loads as well as material properties may vary in time as stationary or non-stationary processes. Time may also be present in the limit state function as an explicit parameter. As a result, the failure probability of a structure is also time dependent. The general formulation for the failure probability for a period of time t may be presented as:

$$P_F(t) = P\,[\,min\; g(\mathbf{x}(\tau);\tau) < 0 \;\; for\; 0 < \tau < t\,] \qquad \text{(B1)}$$

$g(.)$ = limit state function
$\mathbf{x}(\tau)$ = vector of basic variable at time τ
t = period of time under consideration
τ = time

The failure may be of ULS as well as SLS type. One should keep in mind that also in the case of a non-deteriorating time independent resistance and a stationary loading condition, the failure probability is also time dependent due to the random fluctuations of the load. This, however, is usually not considered as a durability problem.

Given (B1), the conditional failure rate (also referred to as risk functions) at time t may be found as:

$$r(t) = \frac{P(failure\;\; in\left[t, t+\Delta t\right] \mid survival\;\; up\;\; t)}{\Delta t} = \frac{p_F(t)}{1-P_F(t)} \qquad \text{(B2)}$$

where

$$p_F(t) = \frac{dP_F(t)}{dt} \qquad \text{(B3)}$$

is the failure time density. For small values of t, the failure probability $P_F(t)$ is close to zero, which makes the conditional failure rate and the density almost numerically equal. For durability problems, the conditional failure rate is usually increasing in time. Reliability limits set in section 7 may be related to (B2) or (B3) whichever is appropriate.

If failure of a structural element leads automatically to replacement by a similar element, one may alternatively use the renewal density h, defined as;

$$h(t) = \frac{\sum_{n=1}^{\infty} P(\text{failure of element number } n \text{ in } [t, t+\Delta t])}{\Delta t} \quad \text{(B4)}$$

For small t the result will be equal to (B2) and (B3). For large t the value of h will asymptotically lead to $1/\mu$ and where μ is the mean time to failure, defined as:

$$\mu = \int_0^{\infty} t \; p_F(t)\, dt = \int_0^{\infty} \left(1 - P_F(t)\right) dt \quad \text{(B5)}$$

The calculation procedure for $P_F(t)$ depends on the nature of the limit state function *g(.)*. If *g(.)* is a smooth monotonically decreasing function not depending explicitly on random process variables, the minimum value is reached at the end of the period, and we simply have:

$$P_F(t) = P\,[g(\mathbf{x};t) < 0\,] \quad \text{(B6)}$$

If *g(.)* depends on random process variables and, therefore, is not monotonically decreasing, we have a first passage problem. In that case the following upper bound approximation may be useful:

$$P_F(t) = P_F(0) + \int_0^t \nu^-(\tau)d\tau \quad \text{(B7)}$$

where $P_F(0)$ is the failure at the start and ν^- the outcrossing rate or unconditional failure rate which is given by:

$$\nu^-(\tau) = \frac{P[g(\tau) > 0 \cap g(\tau + \Delta\tau) < 0]}{\Delta\tau} \quad \text{(B8)}$$

In general, the limit state function *g(.)* may be quite complex due to a combination of physical, chemical and mechanical processes. Take as an example the deterioration processes due to carbonation and/or chloride ingress of concrete. After some period of time the carbonation or chloride fronts may reach the reinforcement and corrosion may start, resulting eventually in spalling and later even in failure by collapse due to some large mechanical load (see figure B1). Many parameters like the outside climate, the cover of the concrete, the diffusion properties, the corrosion speed and so on may play a role.

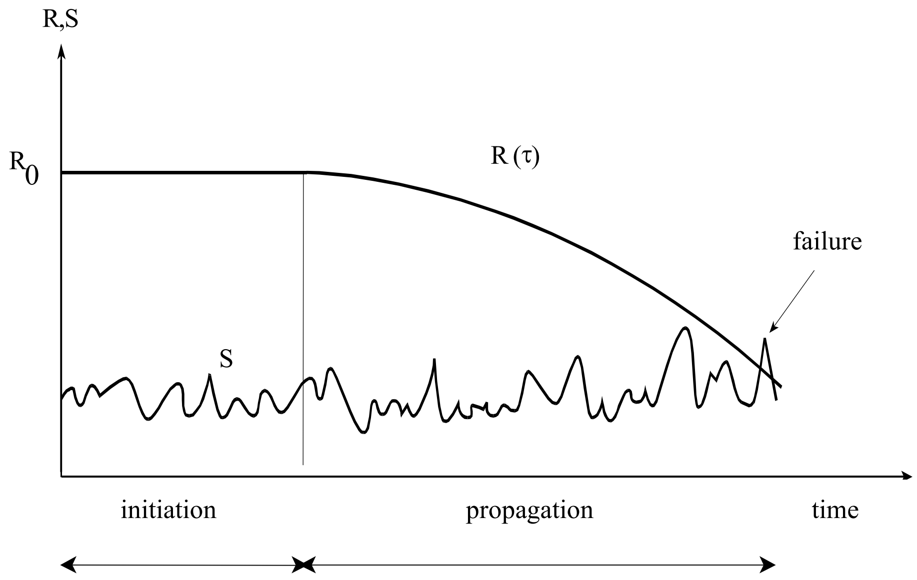

Figure B1: Failure due to a combination of physical and chemical processes and a variable mechanical load

9.2 Modelling of deterioration processes

In this Annex we will restrict the discussion to a family of relatively simple damage accumulation processes that can be described by the following differential equation:

$$\frac{dy}{dt} = y^k \, h(z) \tag{B9}$$

where

y(t) = damage indicator
z(t) = random process of disturbances
h(.) = positive definite function of z
k = parameter determining the nature of the process

From B(9) we may arrive at:

$$\int_{y(0)}^{y(t)} y^{-k} \, dy = \int_0^t h(z(\tau)) \, d\tau \tag{B10}$$

Defining $\Psi(y)$ as the integral function of y^{--k} and $\chi(t)$ as the right hand side integral of (B10), this can be written as:

$$\Psi(y(t)) - \Psi(y(0)) = \chi(t)$$

If z(t) is stationary and ergodic, $\chi(t)$ may asymptotically be taken as implying that the damage increases smoothly:

$$\chi(t) = t\,E\{h(z(t)\} \qquad \text{(B11)}$$

Failure will occur if de damage y(t) exceeds some critical value y_{cr}, which leads finally to the following expression for the limit state function:

$$g(t) = \Psi(\,y_{cr}\,(t)) - \Psi(y(0)) - \chi(t) \qquad \text{(B12)}$$

The critical value y_{cr} may be a constant or time dependent. If y_{cr} is a constant we may use (B3), to find the failure probability. If y_{cr} is time dependent we have a first passage problem.

Characteristic examples

1. *Abrasion / corrosion modelling*

Abrasion and/or corrosion mechanisms can be modelled by k=0 and h(z) = z. In that case (B9) reduces to:

$$\frac{dy}{dt} = z(t)$$

For abrasion or corrosion the damage parameter y corresponds to the thickness of the lost material and z represents is the abrasion or corrosion rate. In this case Ψ is simple equal to y itself. Assuming that z(t) is a stationary and ergodic random process with mean μ_z, we may use (B12) and arrive at:

$$g(t) = y_{cr} - y_o - \mu_z\,t$$

The value y_o may be 0 (or random) and the critical value of y_{cr} may be related to the load and material strength, for instance:

$$y_{cr} = d_o - S/f$$

where d_o is the original material thickness, S the load per unit length and f the material rupture strenght. It can easily be seen that y_{cr} is constant in time for a constant load S and that y_{cr} is time dependent for a fluctuating load.

2. *Duration of load*

We consider again the case n=0 and h(z) = z. Let now, however, y represent the relative reduction of the material strength R, that is $R(t) = R_o(1-y)$.Let further the disturbance z be proportional to the mechanical load S. In other words: the presence of a load will lead to a damage or strength reduction, and more if the load is higher. Such a model can be used to represent duration of load effects. If we define $z = S/S_o$, with S_o some random material parameter, we arrive at:

$$g(t) = y_{cr} - y_o - \mu_S\, t / S_o$$

Let $y_o = 0$ and let y_{cr} correspond to $R(t) = R_o(1- y_{cr}) = S(t)$, we arrive finally at:

$$g(t) = (1- \mu_S\, t / S_o) - S(t)/R_o$$

or equivalently:

$$g(t) = R_o(1- \mu_S\, t / S_o) - S(t)$$

Again, if S is a constant load we may use (B6); if not we have a first passage problem. The resulting time dependent strength for a constant load S is presented in figure B2.

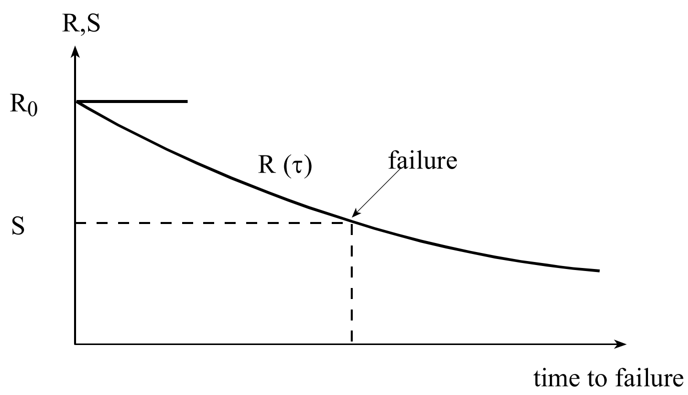

Figure B2: Load duration dependent strength under constant load

3. Fatigue Crack Propagation

Due to load fluctuations some initial small crack in a structure may grow and weaken the cross section. Finally some large load amplitude may lead to collapse of the structural element (see figure B3). The differential equation for the crack growth a is given by:

$$\frac{da}{dn} = C\, Y(a)\, [\, \Delta S(n)\, \sqrt{\pi a}\,]^m$$

Where ΔS represents the stress range, Y(a) represents a geometrical function, C and m are material constants and n is the stress cycle number. Note that in this example the time t has been replaced by the load cycle number n and that k in (B5) corresponds to m/2. The functions Ψ and χ are then given by (assuming ΔS to be stationary and ergodic):

$$\Psi = \frac{2}{2-m}\frac{1}{CY}\, \pi^{--m/2}\, p^{-m/2}\, a^{1-m/2}$$

$$\chi = n\, E\{(\Delta S)^m\}$$

And the limit state function is given by:

$$g(t) = \Psi(a_{cr}) - \Psi(a_0) - \chi$$

where a_0 is the initial crack length and a_{cr} the critical crack length, which again may be time dependent or time independent. In the fist case (B6) may be used, in the second case we have a first passage problem.

Alternatively, one may formulate the limit state function in the crack domain:

$$g(t) = a_{cr} - a(n) \qquad \text{with } a(n) = \left\{ a_o^{1-m/2} + \frac{2}{2-m} C\, Y\, \pi^{m/2}\, nE\{\Delta s^m\} \right\}$$

or in the time domain:

$$g(t) = N - n \qquad \text{with } N = \frac{\psi(a_{cr}) - \psi(a_o)}{E\{(\Delta S)^m\}}$$

These alternative formulations are fully equivalent to the first one.

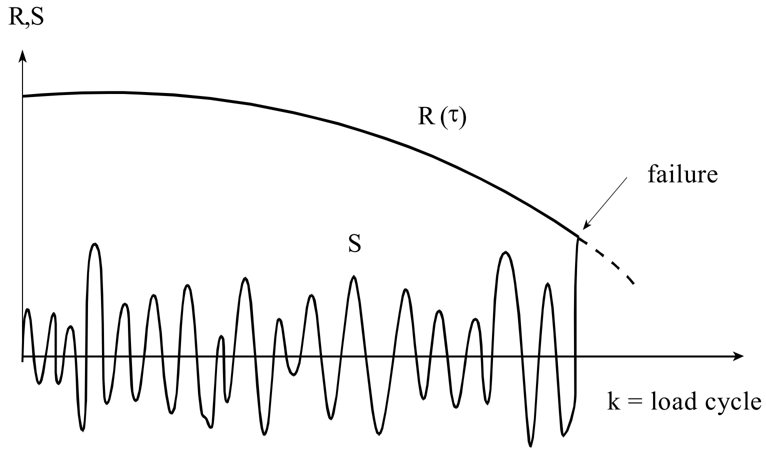

Figure B3: Fatigue fracture under cyclic loading

9.2. Effect of inspection

In the case of deteriorating processes it may be uneconomic to design a structure in such a way that the reliability is sufficient for a normal design life of 50 years. In those cases a more economical solution can be obtained by the definition of an inspection scheme. In those cases failure will not occur if the inspection reveals some predefined deterioration criterion and the structure is repaired adequately.

The sequence of events can be represented in an event tree as indicated in Figure B4. Let the first inspection I_i be planned at time t_i. In that case we may have three possibilities.

1) a failure occurs before t_i (branche F)
2) the inspection detects a serious defect and repair is necessary (branche R)
3) no serious defect is detected and a next inspection at $t = t_2$ is planned

If the structure is repaired, one may usually assume that all variables are reset to the initial situation. From every event R then a new event tree of the same type as the one in figure B4 is started.

For reasons of simplicity we will start by having one inspection only. Using the total probability theorem, the probability of failure for a period t may then formally be written as:

$$P_F(t) = P[\ F \mid Z_i > 0\ ]\ P(Z_i > 0) + P[\ F \mid Z_i < 0\ ]\ P(Z_i < 0) \qquad \text{(B13)}$$

where

F = failure

Z_i = inspection result of inspection at time t_i (negative values correspond to the detection of damage)

If we assume that in the case of a serious damage revealed at the inspection (that is Z<0) the structure will be repaired adequately, (B13) may be reduced to (replacing F by $\min_\tau g(\tau) < 0$, where g( ) is the limit state function and $0 < \tau < t$):

$$P_F(t) = P[\ \min_\tau g(\tau) < 0 \mid Z_i > 0\ ]\ P(Z_i > 0)\ ]$$

or simply:

$$P_F(t) = P[\ \min_\tau g(\tau) < 0 \ \cap\ Z_i > 0\ ]$$

If more inspections in fixed intervals are present we arrive at:

$$P_F(t) = P\ [\ \min_\tau g(\mathbf{x}(\tau);\tau) < 0 \ \cap\ \{\cap Z_i(\mathbf{x}(t_i);t_i)\} > 0 \ \text{ for } 0 < \tau < t\ ] \qquad \text{(B14)}$$

t_i = time of inspection; only inspections with $t_i < \tau$ are relevant

Note: whether or not an inspection is planned, of course, is a matter of economy.

9.3. Example

Figure B5 clarifies formula (B14) for the case of fatigue. As discussed before, the g-function for the situation at the load cycle at time τ is given by:

$$g = a_{cr} - a(t)$$

Let the crack $a(\tau)$ be monitored by a yearly inspection. If the measured crack a_m is larger that some limit a_{lim} the structure will be adequately repaired. An inspection failure may then be modelled as $Z_{ins} < 0$ with:

$$Z_{ins} = a_{lim} - a_m\ (t_i)$$

In present practice a_{lim} usually corresponds to the detection limit and the probability distribution for a_{lim} is then equal to the so called POD-curve (probability of detection).

Failure will occur only if the measured value $a_m(t_{ins})$ is below the limit value a_{lim} at inspection t_i but above the a_{crit} before the next inspection. This way failure probability can be reduced by shorter inspection intervals or by more refined or accurate inspection techniques.

Note that an implication of this method is that these Probability of Detection curves (POD curves) and measurement accuracy's must be known to the designer in order to decide whether or not a certain structure meets the reliability requirements. Note further that the probability of repair is given by:

$$P = P[Z_{ins} < 0]$$

Repair may be considered like some serviceability limit state. The designer should also make sure that the probability of repair is below some economic limit value.

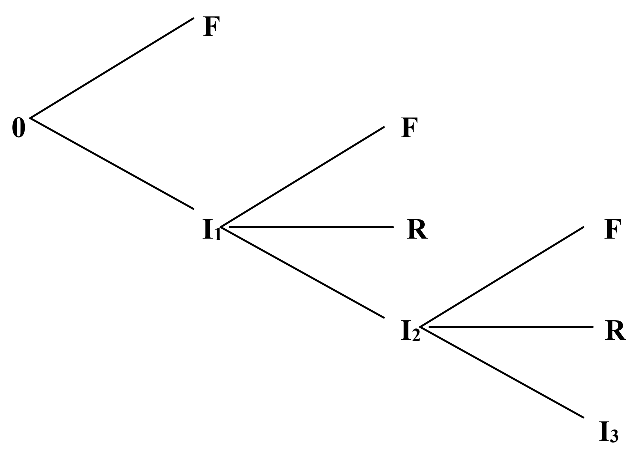

Figure B4: Event tree representation of an inspected component: R = Repair or maintenance action;
F = Failure, I_i = Inspection at time t_i

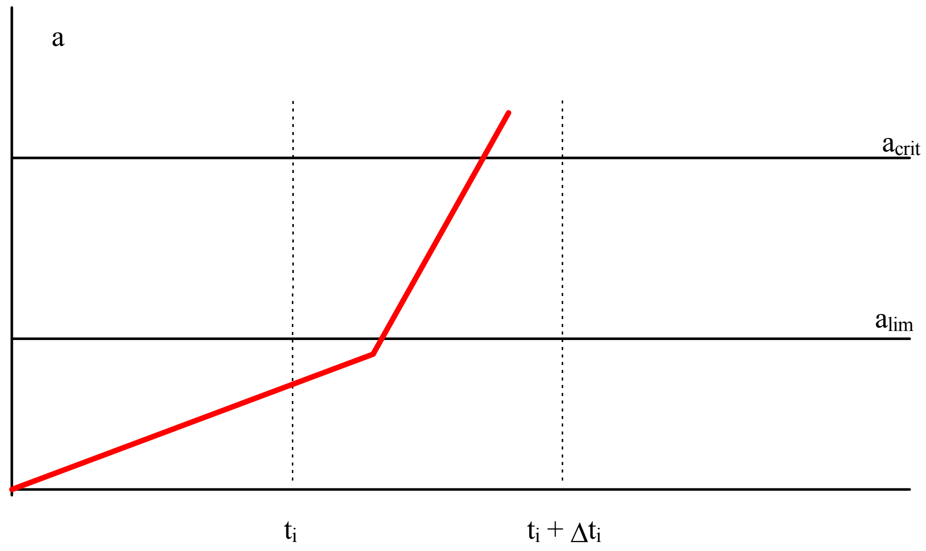

Figure B5: Fatigue failure in the interval t_i, $t_i + \Delta t_i$ with $a(\tau) < a_{lim}$ at the beginning of the interval.

10. Annex C: Reliability Analysis Principles

10.1. Introduction

In recent years, practical reliability methods have been developed to help engineers tackle the analysis, quantification, monitoring and assessment of structural risks, undertake sensitivity analysis of inherent uncertainties and make appropriate decisions about the performance of a structure. The structure may be at the design stage, under construction or in actual use.

This Annex C summarizes the principles and procedures used in formulating and solving risk related problems via reliability analysis. It is neither as broad nor as detailed as available textbooks on this subject, some of which are included in the bibliography. Its purpose is to underpin the updating and decision-making methodologies presented in part 2 of this document.

Starting from the principles of limit state analysis and its application to codified design, the link is made between unacceptable performance and probability of failure. It is important, especially in assessment, to distinguish between components and systems. System concepts are introduced and important results are summarized. The steps involved in carrying out a reliability analysis, whose main objective is to estimate the failure probability, are outlined and alternative techniques available for such an analysis are presented. Some recommendations on formulating stochastic models for commonly used variables are also included.

10.2. Concepts

10.2.1. Limit States

The structural performance of a whole structure or part of it may be described with reference to a set of limit states which separate acceptable states of the structure from unacceptable states. The limit states are divided into the following two categories:

- ultimate limit states, which relate to the maximum load carrying capacity.
- serviceability limit states, which relate to normal use.

The boundary between acceptable (safe) and unacceptable (failure) states may be distinct or diffuse but, at present, deterministic codes of practice assume the former.Thus, verification of a structure with respect to a particular limit state is carried out via a model describing the limit state in terms of a function (called the limit state function) whose value depends on all relevant design parameters. In general terms, attainment of the limit state can be expressed as:

$$g(\mathbf{s}, \mathbf{r}) = 0 \qquad \text{(C.1)}$$

where **s** and **r** represent sets of load (actions) and resistance variables. Conventionally, $g(\mathbf{s}, \mathbf{r}) \leq 0$ represents failure; in other words, an adverse state.

The limit state function, $g(\mathbf{s}, \mathbf{r})$, can often be separated into one resistance function, *r(.)*, and one loading (or action effect) function, *s(.)*, in which case equation (C.) can be expressed as:

$$r(\mathbf{r}) - s(\mathbf{s}) = 0 \qquad \text{(C.2)}$$

10.2.2. Structural Reliability

Load, material and geometry parameters are subject to uncertainties, which can be classified according to their nature, see section 3. They can, thus, be represented by random variables (this being the simplest possible probabilistic representation, whereas more advanced models might be appropriate in certain situations, such as random fields). The variables **S** and **R** are often referred to as "basic random variables" (where the upper case letter is used for denoting random variables) and may be collectively represented by a random vector **X**.

In this context, failure is a probabilistic event and its probability of occurrence, P_f, is given by:

$$P_f = \text{Prob}\ \{ g(\mathbf{X}) \leq 0 \} = \text{Prob}\ \{ \mathrm{M} \leq 0 \} \qquad \text{(C.3a)}$$

where, $\mathrm{M} = g(\mathbf{X})$. Note that M is also a random variable, called the safety margin.

If the limit state function is expressed in the form of eqn (C.2), eqn (C.3a) can be written as

$$P_f = \text{Prob}\ \{ r(\mathbf{R}) \leq s(\mathbf{S}) \} = \text{Prob}\ \{ \mathrm{R} \leq \mathrm{S} \}$$

where $\mathrm{R} = r(\mathbf{R})$ and $\mathrm{S} = s(\mathbf{S})$ are random variables associated with resistance and loading respectively. This expression is useful in the context of the discussion in section 2.2 on code formats and partial safety factors but will not be further used herein.

The failure probability defined in eqn (A.5a) can also be expressed as follows:

$$P_f = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x} \qquad \text{(C.3b)}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function of **X**.
The reliability, P_s, associated with the particular limit state considered is the complementary event, i.e.

$$P_s = 1 - P_f \tag{C.4}$$

In recent years, a standard reliability measure, the reliability index β, has been adopted which has the following relationship with the failure probability

$$\beta = -\Phi^{-1}(P_f) = \Phi^{-1}(P_s) \tag{C.5}$$

where $\Phi^{-1}(.)$ is the inverse of the standard normal distribution function, see Table A.1.

Table C.1: Relationship between β and P_f

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1.3	2.3	3.1	3.7	4.2	4.7	5.2

In most engineering applications, complete statistical information about the basic random variables **X** is not available and, furthermore, the function $g(.)$ is a mathematical model which idealizes the limit state. In this respect, the probability of failure evaluated from eqn (C.3a) or (C.3b) is a point estimate given a particular set of assumptions regarding probabilistic modelling and a particular mathematical model for $g(.)$. The uncertainties associated with these models can be represented in terms of a vector of random parameters **Q**, and hence the limit state function may be re-written as $g(\mathbf{X}, \mathbf{Q})$. It is important to note that the nature of uncertainties represented by the basic random variables **X** and the parameters **Q** is different. Whereas uncetainties in **X** cannot be influenced without changing the physical characteristics of the problem (e.g. changing the steel grade), uncertainties in **Q** can be influenced by the use of alternative methods and collection of additional data.

In this context, eqn (C.3b) may be recast as follows

$$P_f(\theta) = \int_{g(\mathbf{x},\theta) \leq 0} f_{\mathbf{X}|\Theta}(\mathbf{x}|\theta)\, d\mathbf{x} \tag{C.6}$$

where $P_f(\theta)$ is the conditional probability of failure for a given set of values of the parameters θ and $f_{\mathbf{X}|\theta}(\mathbf{x}|\theta)$ is the conditional probability density function of **X** for given θ.

In order to account for the influence of parameter uncertainty on failure probability, one may evaluate the expected value of the conditional probability of failure, i.e.

$$\overline{P_f} = \mathrm{E}\,[P_f(\theta)] = \int_{\theta} P_f(\theta)\, f_{\Theta}(\theta)\, d\theta \qquad \text{(C.7a)}$$

where $f_{\theta}(\theta)$ is the joint probability density function of θ. The corresponding reliability index is given by

$$\overline{\beta} = -\,\Phi^{-1}(\overline{P_f}) \qquad \text{(C.7b)}$$

The main objective of reliability analysis is to estimate the failure probability (or, the reliability index). Hence, it replaces the deterministic safety check with a probabilistic assessment of the safety of the structure, e.g. eqn (C.3) or eqn (C.7). Depending on the nature of the limit state considered, the uncertainty sources and their implications for probabilistic modeling, the characteristics of the calculation model and the degree of accuracy required, an appropriate methodology has to be developed. In many respects, this is similar to the considerations made in formulating a methodology for deterministic structural analysis but the problem is now set in a probabilistic framework.

10.2.3. System Concepts

Structural design is, at present, primarily concerned with component behaviour. Each limit state equation is, in most cases, related to a single mode of failure of a single component.

However, most structures are an assembly of structural components and even individual components may be susceptible to a number of possible failure modes. In deterministic terms, the former can be tackled through a progressive collapse analysis (particularly appropriate in redundant structures), whereas the latter is usually dealt with by checking a number of limit state equations.

However, the system behaviour of structures is not well quantified in limit state codes and requires considerable innovation and initiative from the engineer. A probabilistic approach provides a better platform from which system behaviour can be explored and utilised. This can be of benefit in assessment of existing structures where strength reserves due to system effects can alleviate the need for expensive strengthening.

There are two fundamental systems, see Fig. C.1:

(1) A series system is a system which fails if one or more of its components fail.
(2) A parallel system is a system which fails when all its components have failed.

The probability of system failure is given by

$$P_{f,\,sys} = P[E_1 \cup E_2 \cup ... \cup E_n] \quad \text{for a series system} \qquad (C.8a)$$
$$P_{f,\,sys} = P[E_1 \cap E_2 \cap ... \cap E_n] \quad \text{for a parallel system} \qquad (C.8b)$$

where E_i (i=1, ...n) is the event corresponding to failure of the ith component. In the case of parallel systems, which are designed to provide some redundancy, it is important to define the state of the component after failure. In structures, this can be described in terms of a characteristic load-displacement response, see Fig. C.2, for which two convenient idealisations are the 'brittle' and the 'fully ductile' case. Intermediate, often more realistic, cases can also be defined.

The above expressions can be difficult to evaluate in the case of large systems with stochastically dependent components and, for this reason, upper and lower bounds have been developed, which may be used in practical applications. In order to appreciate the effect of system behaviour on failure probabilities, results for two special systems comprising equally correlated components with the same failure probability for each component are shown in Fig. C.3(a) and C.3(b). Note that in the case of the parallel system, it is assumed that the components are fully ductile.

More general systems can be constructed by combining the two fundamental types. It is fair to say that system methods are more developed for skeletal rather than continuous structures. Important results from system reliability theory are summarized in section 4.

10.3. Component Reliability Analysis

The framework for probabilistic modeling and reliability evaluation is outlined in this section. The focus is on the procedure to be followed in assessing the reliability of a critical component with respect to a particular failure mode.

10.3.1. General Steps

The main steps in a component reliability analysis are the following:

(1) select appropriate limit state function
(2) specify appropriate time reference
(3) identify basic variables and develop appropriate probabilistic models
(4) compute reliability index and failure probability

(5) perform sensitivity studies

Step (1) is essentially the same as for deterministic analysis. Step (2) should be considered carefully, since it affects the probabilistic modeling of many variables, particularly live loading. Step (3) is perhaps the most important because the considerations made in developing the probabilistic models have a major effect on the results obtained, see section 3.2. Step (4) should be undertaken with one of the methods summarized in section 3.3, depending on the application. Step (5) is necessary insofar as the sensitivity of any results (deterministic or probabilistic) should be assessed before a decision is taken.

10.3.2. Probabilistic Modelling

For the particular failure mode under consideration, uncertainty modeling must be undertaken with respect to those variables in the corresponding limit state function whose variability is judged to important (basic random variables). Most engineering structures are affected by the following types of uncertainty:

- intrincic physical or mechanical uncertainty; when considered at a fundamental level, this uncertainty source is often best described by stochastic processes in time and space, although it is often modelled more simply in engineering applications through random variables.
- measurement uncertainty; this may arise from random and systematic errors in the measurement of these physical quantities
- statistical uncertainty; due to reliance on limited information and finite samples
- model uncertainty; related to the predictive accuracy of calculation models used

The physical uncertainty in a basic random variable is represented by adopting a suitable probability distribution, described in terms of its type and relevant distribution parameters. The results of the reliability analysis can be very sensitive to the tail of the probability distribution, which depends primarily on the type of distribution adopted. A proper choice of distribution type is therefore important.

For most commonly encountered basic random variables there have been studies (of varying detail) that contain guidance on the choice of distribution and its parameters. If direct measurements of a particular quantity are available, then existing, so-called *a priori*, information (e.g. probabilistic models found in published studies) should be used as prior statistics with a relatively large equivalent sample size ($n' \approx 50$).

The following comments may also be helpful in selecting a suitable probabilistic model.

Material properties

- frequency of negative values is normally zero
- log-normal distribution can often be used
- distribution type and parameters should, in general, be derived from large homogeneous samples and with due account of established distributions for similar variables (e.g. for a new high strength steel grade, the information on properties of existing grades should be consulted); tests should be planned so that they are, as far as possible, a realistic description of the potential use of the material in real applications.

Geometric parameters

- variability in structural dimensions and overall geometry tends to be small
- dimensional variables can be adequately modelled by the normal or log-normal distribution
- if the variable is physically bounded, a truncated distribution may be appropriate (e.g. location of reinforcement); such bounds should always be carefully considered to avoid entering into physically inadmissible ranges
- variables linked to manufacturing can have large coefficients of variation (e.g. imperfections, misalignments, residual stresses, weld defects).

Load variables

- loads should be divided according to their time variation (permanent, variable, accidental)
- in certain cases, permanent loads consist of the sum of many individual elements; they may be represented by a normal distribution
- for single variable loads, the form of the point-in-time distribution is seldom of immediate relevance; often the important random variable is the magnitude of the largest extreme load that occurs during a specified reference period for which the probability of failure is calculated (e.g. annual, lifetime)
- the probability distribution of the largest extreme could be approximated by one of the asymptotic extreme-value distributions (Gumbel, sometimes Frechet)
- when more than one variable loads act in combination, load modelling is often undertaken using simplified rules suitable for FORM/SORM analysis.

In selecting a distribution type to account for physical uncertainty of a basic random variable afresh, the following procedure may be followed:

- based on experience from similar type of variables and physical knowledge, choose a set of possible distributions
- obtain a reasonable sample of observations ensuring that, as far as possible, the sample points are from a homogeneous group (i.e. avoid systematic variations within the sample) and that the sampling reflects potential uses and applications

- evaluate by an appropriate method the parameters of the candidate distributions using the sample data; the method of maximum likelihood is recommended but evaluation by alternative methods (moment estimates, least-square fit, graphical methods) may also be carried out for comparison.
- compare the sample data with the resulting distributions; this can be done graphically (histogram vs. pdf, probability paper plots) or through the use of goodness-of-fit tests (Chi-square, Kolmogorov-Smirnov tests)

If more than one distributions give equally good results (or if the goodness-of-fit tests are acceptable to the same significance level), it is recommended to choose the distribution that will result in the smaller reliability. This implies choosing distributions with heavy left tails for resistance variables (material properties, geometry excluding tolerances) and heavy right tails for loading variables (manufacturing tolerances, defects and loads).

Capturing the essential features of physical uncertainty in a load or in a structure property through a random variable model is perhaps the simplest way of modeling uncertainty and quantifying its effect on failure probability. In general, loads are functions of both time and position on any particular structure. Equally, material properties and dimensions of even a single structural member, e.g. a RC floor slab, are functions which vary both in time and in space. Such random functions are usually denoted as random (or stochastic) processes when time variation is the most important factor and as random fields when spatial variation is considered.

Fig. C.4(a) shows schematically a continuous stochastic process, e.g. wind pressure at a particular point on a wall of a structure. The trace of this process over time is obtained through successive realisations of the underlying phenomenon, in this case wind speed, which is clearly a random variable taking on different values within each infinitesimally small time interval, δt.
Fig. C.4(b) depicts a two-dimensional random field, e.g. the spatial variation of concrete strength in a floor slab just after construction. Once again, a random variable, in this case describing the possible outcomes of, say, a core test obtained from any given small area, δA, is the basic kernel from which the random field is built up.

In considering either a random process or a random field, it is clear that, apart from the characteristics associated with the random variable describing uncertainty within a small unit (interval or area), laws describing stochastic dependence (or, in simpler terms, correlation) between outcomes in time and/or in space are very important.

The other three types of uncertainty mentioned above (measurement, statistical, model) also play an important role in the evaluation of reliability. As mentioned in section 2.3, these uncertainties are influenced by the particular method used in, for example, strength analysis and by the collection of additional (possibly, directly obtained) data. These uncertainties could be rigorously analysed by adopting the approach outlined by eqns (C.8) and (C.9). However, in many practical applications a simpler approach has been adopted insofar as model (and measurement) uncertainty is concerned based on the differences between results predicted by the mathematical model adopted for $g(\mathrm{x})$ and some more elaborate model believed to be a closer representation of actual conditions. In such cases, a model uncertainty basic random variable X_m is introduced where

$$X_m = \frac{\text{actual value}}{\text{predicted value}}$$

and the following comments offer some general guidance in estimating the statistics of X_m:

- the mean value of the model uncertainty associated with code calculation models can be larger than unity, reflecting the in-built conservatism of code models
- the model uncertainty parameters of a particular calculation model may be evaluated vis-a-vis physical experiments or by comparing the calculation model with a more detailed model (e.g. finite element model)
- when experimental results are used, use of measured rather than nominal or characteristic quantities is preferred in calculating the predicted value
- the use of numerical experiments (e.g. finite element models) has some advantages over physical experiments, since the former ensure well-controlled input.
- the choice of a suitable probability distribution for X_m is often governed by mathematical convenience and a normal distribution has been used extensively.

10.3.3. Computation of Failure Probability

As mentioned above, the failure probability of a structural component with respect to a single failure mode is given by

$$P_f = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x})\, \mathrm{d}\mathbf{x} \qquad \text{(C.3b)}$$

where **X** is the vector of basic random variables, $g(x)$ is the limit state (or failure) function for the failure mode considered and $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function of **X**.

An important class of limit states are those for which all the variables are treated as time independent, either by neglecting time variations in cases where this is considered acceptable or by transforming time-dependent processes into time-invariant variables (e.g. by using extreme value distributions). The methods commonly used for calculating P_f in such cases are outlined below. Guidelines on how to deal with time-dependent problems are given in section 5. Note that after calculating P_f via one of the methods outlined below, or any other valid method, a reliability index may be obtained from equation (C.5), for comparative or other purposes.

Asymptotic approximate methods

Although these methods first emerged with basic random variables described through 'second-moment' information (i.e. with their mean value and standard deviation, but without assigning any probability distributions), it is nowadays possible in many cases to have a full description of the random vector **X** (as a result of data collection and probabilistic modelling studies). In such cases, the probability of failure could be calculated via first or second order reliability methods (FORM and SORM respectively). Their implementation relies on:

(1) Transformation techniques:

$$\mathbf{T}: \quad \mathbf{X} = (X_1, X_2, \ldots X_n) \quad \rightarrow \quad \mathbf{U} = (U_1, U_2, \ldots U_n) \qquad \text{(C.9)}$$

where $U_1, U_2, \ldots U_n$ are independent standard normal variables (i.e. with zero mean value and unit standard deviation). Hence, the basic variable space (including the limit state function) is transformed into a standard normal space, see Fig. C.5. The special properties of the standard normal space lead to several important results, as discussed below.

(2) Search techniques:

In standard normal space, the objective is to determine a suitable checking point: this is shown to be the point on the limit-state surface which is closest to the origin, the so-called 'design point'. In this rotationally symmetric space, it is the most likely failure point, in other words its co-ordinates define the combination of variables that are most likely to cause failure. This is because the joint standard normal density function, whose bell-shaped peak lies directly above the origin, decreases exponentially as the distance from the origin increases. To determine this point, a search procedure is required in all but the most simple of cases (the Rackwitz-Fiessler algorithm is commonly used).

Denoting the co-ordinates of this point by

$$\mathbf{u}^* = (u_1^*, u_2^*, \ldots u_n^*)$$

its distance from the origin is clearly equal to

$$(\sum_{i=1}^{n} u_i^{*2})^{1/2}$$

This scalar quantity is known as the Hasofer-Lind reliability index, β_{HL}, i.e.

$$\beta_{HL} = (\sum_{i=1}^{n} u_i^{*2})^{1/2} \qquad \text{(C.10)}$$

Note that $\mathbf{u}^*$ can also be written as

$$\mathbf{u}^* = \beta_{HL}\, \boldsymbol{\alpha} \qquad \text{(C.11a)}$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots \alpha_n)$ is the unit normal vector to the limit state surface at $\mathbf{u}^*$, and, hence, α_i (i=1,...n) represent the direction cosines at the design point. These are also known as the sensitivity factors, as they provide an indication of the relative importance of the uncertainty in basic random variables on the computed reliability. Their absolute value ranges between zero and unity and the closer this is to the upper limit, the more significant the influence of the respective random variable is to the reliability. The following expression is valid for independent variables

$$\sum_{i=1}^{n} \alpha_1^2 = 1 \qquad \text{(C.11b)}$$

(3) Approximation techniques:

Once the checking point is determined, the failure probability can be approximated using results applicable to the standard normal space. Thus, in a first-order approximation, the limit state surface is approximated by its tangent hyperplane at the design point. The probability content of the failure set is then given by

$$P_{f\text{FORM}} = \Phi(-\beta_{\text{HL}}) \qquad \text{(C.12a)}$$

In some cases, a higher order approximation of the limit state surface at the design point is merited, if only to check the accuracy of FORM. The result for the probability of failure assuming a quadratic (second-order) approximation of the limit state surface is asymptotically given by

$$P_{fSORM} = \Phi(-\beta_{HL}) \prod_{j=1}^{n-1} (1 - \beta_{HL}\kappa_j)^{-1/2} \qquad \text{(C.12b)}$$

for $\beta_{\text{HL}} \rightarrow \infty$, where κ_j are the principal curvatures of the limit state surface at the design point. An expression applicable to finite values of β_{HL} is also available.

Simulation Methods

In this approach, random sampling is employed to simulate a large number of (usually numerical) experiments and to observe the result. In the context of structural reliability, this means, in the simplest approach, sampling the random vector **X** to obtain a set of sample values. The limit state function is then evaluated to ascertain whether, for this set, failure (i.e. $g(\mathbf{x}) \leq 0$) has occurred. The experiment is repeated many times and the probability of failure, P_f, is estimated from the fraction of trials leading to failure divided by the total number of trials. This so-called Direct or Crude Monte Carlo method is not likely to be of use in practical problems because of the large number of trials required in order to estimate with a certain degree of confidence the failure probability. Note that the number of trials increases as the failure probability decreases. Simple rules may be found, of the form $N > C/P_f$, where N is the required sample size and C is a constant related to the confidence level and the type of function being evaluated.

Thus, the objective of more advanced simulation methods, currently used for reliability evaluation, is to reduce the variance of the estimate of P_f. Such methods can be divided into two categories, namely indicator function methods and conditional expectation methods.

An example of the former is Importance Sampling, where the aim is to concentrate the distribution of the sample points in the vicinity of likely failure points, such as the design point obtained from FORM/SORM analysis. This is done by introducing a sampling function, whose choice would depend on *a priori* information available, such as the co-ordinates of the design point and/or any estimates of the failure probability. In this way, the success rate (defined here as a probability of obtaining a result in the failure region in any particular trial) is improved compared to Direct Monte Carlo. Importance Sampling is often used following an initial FORM/SORM analysis. A variant of this method is Adaptive Sampling, in which the sampling density is updated as the simulation proceeds. Importance Sampling could be performed in basic variable or standard normal space, depending on the problem and the form of prior information.

A powerful method belonging to the second category is Directional Simulation. It achieves variance reduction using conditional expectation in the standard normal space, where a special result applies pertaining to the probability bounded by a hypershere centred at the origin. Its efficiency lies in that each random trial generates precise information on where the boundary between safety and failure lies. However, the method does generally require some iterative calculations. It is particularly suited to problems where it is difficult to identify 'important' regions (perhaps due to the presence of multiple local design points).

The two methods outlined above have also been used in combination, which indicates that when simulation is chosen as the basic approach for reliability assessment, there is scope to adapt the detailed methodology to suit the particular problem in hand.

10.3.4. Recommendations

As with any other analysis, choosing a particular method must be justified through experience and/or verification. Experience shows that FORM/SORM estimates are adequate for a wide range of problems. However, these approximate methods have the disadvantage of not being quantified by error estimates, except for few special cases. As indicated, simulation may be used to verify FORM/SORM results, particularly in situations where multiple design points might be suspected. Simulation results should include the variance of the estimated probability of failure, though good estimates of the variance could increase the computations required. When using FORM/SORM, attention should be given to the ordering of dependent random variables and the choice of initial points for the search algorithm. Not least, the results for the design point should be assessed to ensure that they do not contradict physical reasoning.

10.4. System Reliability Analysis

As discussed in section 3, individual component failure events can be represented by failure boundaries in basic variable or standard normal space. System failure events can be similarly represented, see Fig. C.6(a) and C.6(b), and, once more, certain approximate results may be derived as an extension to FORM/SORM analysis of individual components. In addition, system analysis is sometimes performed using bounding techniques and some relevant results are given below.

10.4.1. Series systems

The probability of failure of a series system with *m* components is defined as

$$P_{f\,sys} = \mathrm{P}\left[\bigcup_{j=1}^{m} \mathrm{F}_j\right] \qquad \text{(C.13)}$$

where, F_j is the event corresponding to the failure of the *j*th component. By describing this event in terms of a safety margin M_j

$$P[F_j] = P[M_j \leq 0] \approx \Phi(-\beta_j) \qquad \text{(C.14)}$$

where β_j is its corresponding FORM reliability index, it can be shown that in a first-order approximation

$$P_{f\,sys} = 1 - \Phi_\mathrm{m}\left[\tilde{\beta}\,;\,\tilde{\tilde{\rho}}\right] \qquad \text{(C.15a)}$$

where $\Phi_\mathrm{m}[.]$ is the multi-variate standard normal distribution function, $\tilde{\beta}$ is the (*m x 1*) vector of component reliability indices and $\tilde{\tilde{\rho}}$ is the (*m x m*) correlation matrix between safety margins with elements given by

$$\rho_{jk} = \sum_{i=1}^{n} \alpha_{ij}\alpha_{ik} \qquad ,j,\ k = 1, 2, ...,\mathrm{m} \qquad \text{(C.15b)}$$

where α_{ij} is the sensitivity factor corresponding to the *i*th random variable in the *j*th margin.

In some cases, especially when the number of components becomes large, evaluation of equation (C.15) becomes cumbersome and bounds to the system failure probability may prove sufficient.

Simple first-order linear bounds are given by

$$\underset{j=1}{\overset{m}{Max}}\left[P(F_j)\right] \leq P_{fsys} \leq Min\left[\left(\sum_{j=1}^{m} P(F_j)\right),1\right] \qquad \text{(C.16a)}$$

(A.20b)

but these are likely to be rather wide, especially for large *m*, in which case second-order linear bounds (Ditlevsen bounds) may be needed. These are given by

$$P[F_1] + \sum_{j=2}^{m} Max\left\{\left[P(F_j) - \sum_{k=1}^{j-1} P(F_j \cap F_k)\right], 0\right\} \le P_{fsys} \le$$
$$P[F_1] + \sum_{j=2}^{m} \left\{P(F_j) - \underset{k<j}{Max}\left[P(F_j \cap F_k)\right]\right\} \quad \text{(C.16b)}$$

The narrowness of these bounds depends in part on the ordering of the events. The optimal ordering may differ between the lower and the upper bound. In general, these bounds are much narrower than the simple first-order linear bounds given by equation (C.16a). The bisections of events may be calculated using a first-order approximation, which appears below in the presentation of results for parallel systems.

10.4.2. Parallel Systems

Following the same approach and notation as above, the failure probability of a parallel system with *m* components is given by

$$P_{fsys} = P\left[\bigcap_{j=1}^{m}(F_j)\right] = P\left[\bigcap_{j=1}^{m}(M_j \le 0)\right] \quad \text{(C.17)}$$

and the corresponding first-order approximation is

$$P_{f\,sys} = \Phi_m\left[-\tilde{\beta}\,;\,\tilde{\tilde{\rho}}\right] \quad \text{(C.18)}$$

Simple bounds are given by

$$0 \le P_{fsys} \le \underset{j=1}{\overset{m}{Min}}\left[P(F_j)\right] \quad \text{(C.19a)}$$

These are usually too wide for practical applications. An improved upper bound is

$$P_{fsys} \le \underset{j,k=1}{\overset{m}{Min}}\left[P(F_j \cap F_k)\right] \quad \text{(C.19b)}$$

The error involved in the first-order evaluation of the intersections, P[$F_j \cap F_k$], is, to a large extent, influenced by the non-linearity of the margins at their respective design points. In order to obtain a better estimate of the intersection probabilities, an improvement on the selection of linearisation points has been suggested.

10.5. Time-Dependent Reliability

10.5.1. General Remarks

Even in considering a relatively simple safety margin for component reliability analysis such as $M = R - S$, where R is the resistance at a critical section in a structural member and S is the corresponding load effect at the same section, it is generally the case that both S and resistance R are functions of time. Changes in both mean values and standard deviations could occur for either *R(t)* or *S(t)*. For example, the mean value of *R(t)* may change as a result of deterioration (e.g. corrosion of reinforcement in an RC bridge implies loss of area, hence a reduction in the mean resistance) and its standard deviation may also change (e.g. uncertainty in predicting the effect of corrosion on loss of area may increase as the periods considered become longer). On the other hand, the mean value of *S(t)* may increase over time (e.g. due to higher traffic flow and/or higher individual vehicle weights) and, equally, the estimate of its standard deviation may increase due to lower confidence in predicting the correct mix of traffic for longer periods. A time-dependent reliability problem could thus be schematically represented as in Fig. C.7, the diagram implying that, on average, the reliability decreases with time. Although this situation is usual, the converse could also occur in reliability assessment of existing structures, for example through strengthening or favourable change in use.

Thus, the elementary reliability problem described through equations (C.3a) and (C.3 b) may now be formulated as:

$$P_f(t) = \text{Prob}\{\, \mathbf{R}(t) \le \boldsymbol{S}(t) \,\} = \text{Prob}\,\{\, g(\mathbf{X}(t)) \le 0 \,\}\ 00{,}0 \qquad \text{(C.20a)}$$

where $g(\mathbf{X}(t)) = \mathrm{M}(t)$ is a time-dependent safety margin, and

$$P_f(t) = \int_{g(\mathbf{x}(t)) \le 0} f_{\mathbf{X}(t)}(\mathbf{x}(t))\, d\mathbf{x}(t) \qquad \text{(C.20b)}$$

is the instantaneous failure probability at time t, assuming that the structure was safe at time less than t.

In time-dependent reliability problems, interest often lies in estimating the probability of failure over a time interval, say from 0 to t_L. This could be obtained by integrating $P_f(t)$ over the interval $[0, t_L]$, bearing in mind the correlation characteristics in time of the process $\mathbf{X}(t)$ - or, sometimes more conveniently, the process $\mathbf{R}(t)$, the process $\mathbf{S}(t)$, as well as any cross correlation between $\mathbf{R}(t)$ and $\mathbf{S}(t)$. Note that the load effect process $\mathbf{S}(t)$ is often composed of additive components, $S_1(t)$, $S_2(t)$, ..., for each of which the time fluctuations may have different features (e.g. continuous variation, pulse-type variation, spikes).

Interest may also lie in predicting when S(t) crosses R(t) for the first time, see Figure C.8, or the probability that such an event would occur within a specified time interval. These considerations give rise to so-called ‘crossing’ problems, which are treated using stochastic process theory. A key concept for such problems is the rate at which a random process X(t) ‘upcrosses’ (or crosses with a positive slope) a barrier or level ξ, as shown in Figure A.9. This upcrossing rate is a function of the joint probability density function of the process and its derivative, and is given by Rice's formula

$$\nu_{\xi}^{+} = \int_{0}^{\infty} \dot{x}\, f_{X\dot{X}}(\xi, \dot{x}\,)\, d\dot{x} \qquad \text{(C.21)}$$

where the rate in general represents an ensemble average at time *t*. For a number of common stochastic processes, useful results have been obtained starting from Equation (C.21). An important simplification can be introduced if individual crossings can be treated as independent events and the occurences may be approximated by a Poisson distribution, which might be a reasonable assumption for certain rare load events.

Another class of problems calling for a time-dependent reliability analysis are those related to damage accumulation, such as fatigue and fracture. This case is depicted in Fig. C.10 via a fixed threshold (e.g. critical crack size) and a monotonically increasing time-dependent load effect or damage function (e.g. actual crack size at any given time).

It is evident from the above remarks that the best approach for solving a time-dependent reliability problem would depend on a number of considerations, including the time frame of interest, the nature of the load and resistance processes involved, their correlation properties in time, and the confidence required in the probability estimates. All these issues may be important in determining the appropriate idealisations and approximations.

10.5.2. Transformation to Time-Independent Formulations

Although time variations are likely to be present in most structural reliability problems, the methods outlined in Sections 3 and 4 have gained wide acceptance, partly due to the fact that, in many cases, it is possible to transform a time dependent failure mode into a corresponding time independent mode. This is especially so in the case of overload failure, where individual time-varying actions, which are essentially random processes, *p(t)*, can be modelled by the distribution of the maximum value within a given reference period T, i.e. $X = \max_T\{\, p(t)\}$ rather than the point-in-time distribution. For continuous processes, the probability distribution of the maximum value (i.e. the largest extreme) is often approximated by one of the asymptotic extreme value distributions. Hence, for structures subjected to a single time-varying action, a random process model is replaced by a random variable model and the principles and methods given previously may be applied.

The theory of stochastic load combination is used in situations where a structure is subjected to two or more time-varying actions acting simultaneously. When these actions are independent, perhaps the most important observation is that it is highly unlikely that each action will reach its peak lifetime value at the same moment in time. Thus, considering two time-varying load processes $p_1(t)$, $p_2(t)$, $0 \leq t \leq T$, acting simultaneously, for which their combined effect may be expressed as a linear combination $p_1(t)+ p_2(t)$, the random variable of interest is:

$$X = \max_T\{ p_1(t)+ p_2(t)\} \tag{C.22a}$$

If the loads are independent, replacing X by $\max_T\{p_1(t)\} + \max_T\{p_2(t)\}$ leads to very conservative results. However, the distribution of X can be derived in few cases only. One possible way of dealing with this problem, which also leads to a relatively simple deterministic code format, is to replace X with the following

$$X' = \max_T \begin{cases} \max_T\{ p_1(t)\} + p_2(t) \\ p_1(t) + \max_T\{ p_2(t)\} \end{cases} \tag{C.22b}$$

This rule (Turkstra's rule) suggests that the maximum value of the sum of two independent load processes occurs when one of the processes attains its maximum value. This result may be generalised for several independent time-varying loads. The conditions which render this rule adequate for failure probability estimation are discussed in standard texts. Note that the failure probability associated with the sum of a special type of independent identically distributed processes (so-called FBC process) can be calculated in a more accurate way, as will be outlined below. Other results have been obtained for combinations of a number of other processes, starting from Rice's barrier crossing formula.

The FBC (Ferry Borges-Castanheta) process is generated by a sequence of independent identically distributed random variables, each acting over a given (deterministic) time interval. This is shown in Fig. C.11 where the total reference period T is made up of n_i repetitions where n_i=T/τ_i. Hence, the FBC process is a rectangular pulse process with changes in amplitude occurring at equal intervals. Because of independence, the maximum value in the reference period T is given by

$$F_{\max_T X_i}(x_i) = [F_{X_i}(x_i)]^{n_i} \qquad (C.23)$$

When a number of FBC processes act in combination and the ratios of their repetition numbers within a given reference period are given by positive integers it is, in principle, possible to obtain the extreme value distribution of the combination through a recursive formula. More importantly, it is possible to deal with the sum of FBC processes by implementing the Rackwitz-Fiessler algorithm in a FORM/SORM analysis.

A deterministic code format, compatible with the above rules, leads to the introduction of combination factors, ψ_{0i}, for each time-varying load i. In principle, these factors express ratios between fractiles in the extreme value and point-in-time distributions so that the probability of exceeding the design value arising from a combination of loads is of the same order as the probability of exceeding the design value caused by one load. For time-varying loads, they would depend on distribution parameters, target reliability and FORM/SORM sensitivity factors and on the frequency characteristics (i.e. the base period assumed for stationary events) of loads considered within any particular combination.

10.5.3. Introduction to Crossing Theory

In considering a time-dependent safety margin, i.e. $M(t) = g(\mathbf{X}(t))$, the problem is to establish the probability that $M(t)$ becomes zero or less in a reference time period, t_L. As mentioned previously, this constitutes a so-called 'crossing' problem. The time at which $M(t)$ becomes less than zero for the first time is called the 'time to failure' and is a random variable, see Fig. C.12(a), or, in a basic variable space, Fig. C.12(b). The probability that $M(t) \leq 0$ occurs during t_L is called the'first-passage' probability. Clearly, it is identical to the probability of failure during time t_L.

The determination of the first passage probability requires an understanding of the theory of random processes. Herein, only some basic concepts are introduced in order to see how the methods described above have to be modified in dealing with crossing problems.
The first-passage probability, $P_f(t)$ during a period $[0, t_L]$ is

$$P_f(t_L) = 1 - P[\, N(t_L)=0 \mid \mathbf{X}(0) \in D]\, P[\mathbf{X}(0) \in D] \qquad (C.24a)$$

where $\mathbf{X}(0) \in D$ signifies that the process $\mathbf{X}(t)$ starts in the safe domain and $N(t_L)$ is the number of outcrossings in the interval $[0, t_L]$. The second probability term is equivalent to $1 - P_f(0)$, where $P_f(0)$ is the probability of failure at t=0. Equation (C.28a) can be re-written as

$$P_f(t_L) = P_f(0) + (1 - P_f(0))\,(1 - P[N(t_L)=0]) \qquad \text{(C.24b)}$$

from which different approximations may be derived depending on the relative magnitude of the terms. A useful bound is

$$P_f(t_L) \leq P_f(0) + E[N(t_L)] \qquad \text{(C.25)}$$

where the first term may be calculated by FORM/SORM and the expected number of outcrossings, $E[N(t_L)]$, is calculated by Rice's formula or one of its generalisations. Alternatively, parallel system concepts can be employed.

10.6. Figures

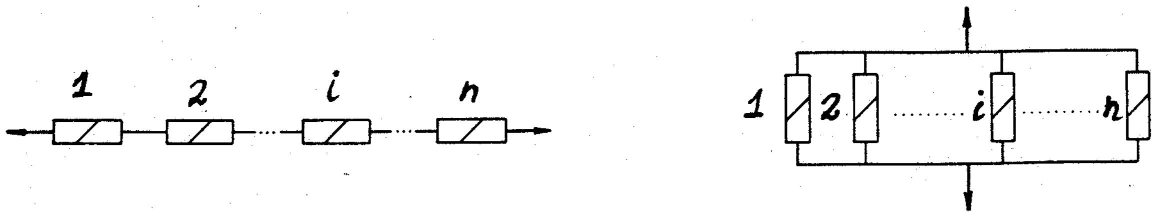

Figure C.1: Schematic representation of series and parallel systems

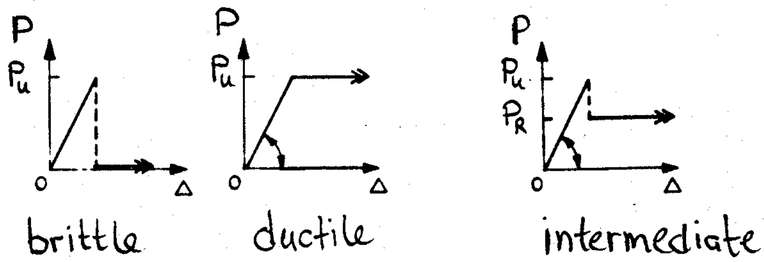

Figure C.2: Idealised load-displacement response of structural elements

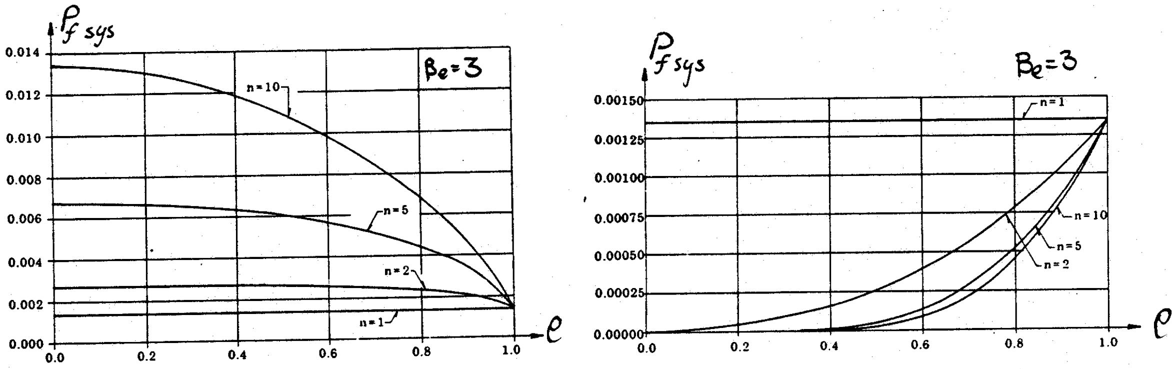

Figure C.3: Effect of element correlation and system size on failure probability
(a) series system (b) parallel system

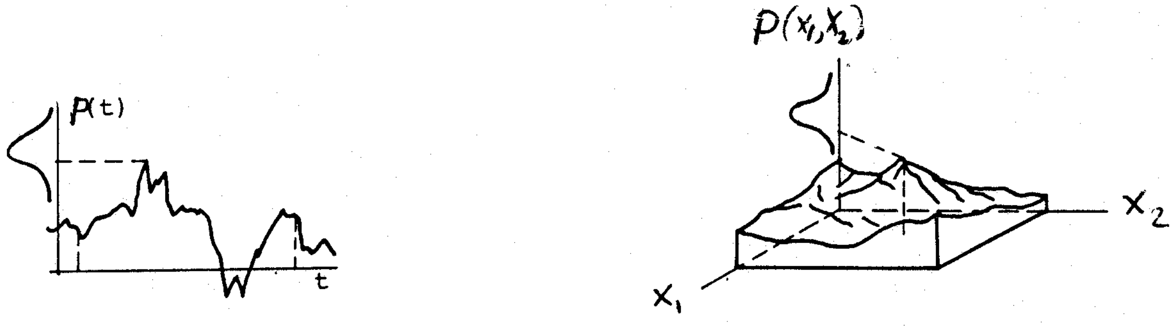

Figure C.4: Schematic representations
(a) random process (b) random field

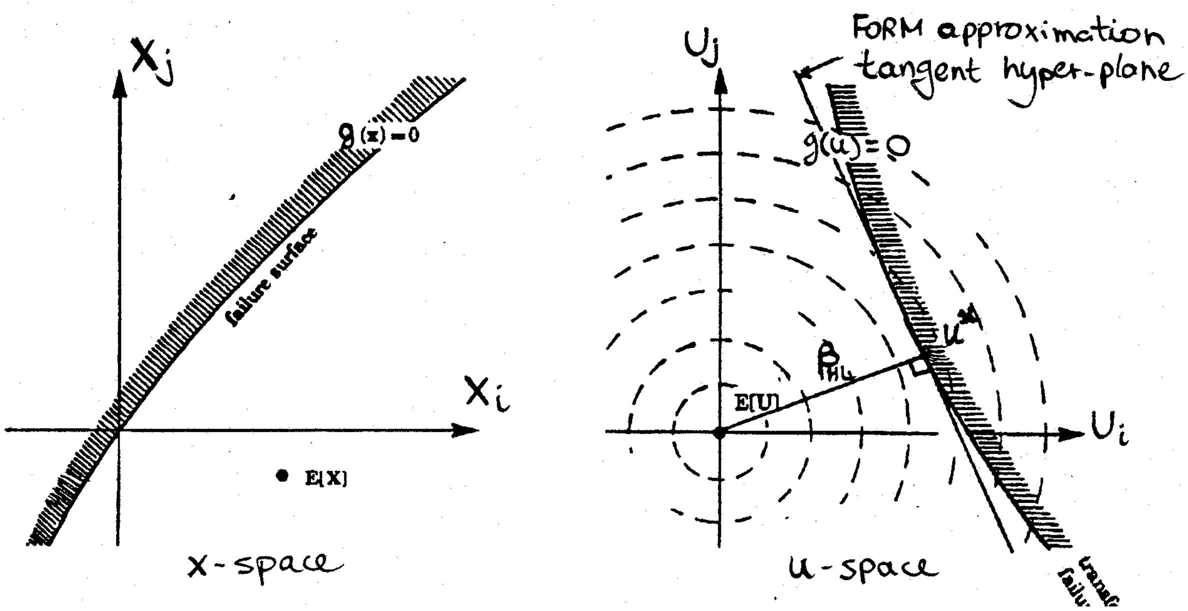

Figure C.5: Limit state surface in basic variable and standard normal space

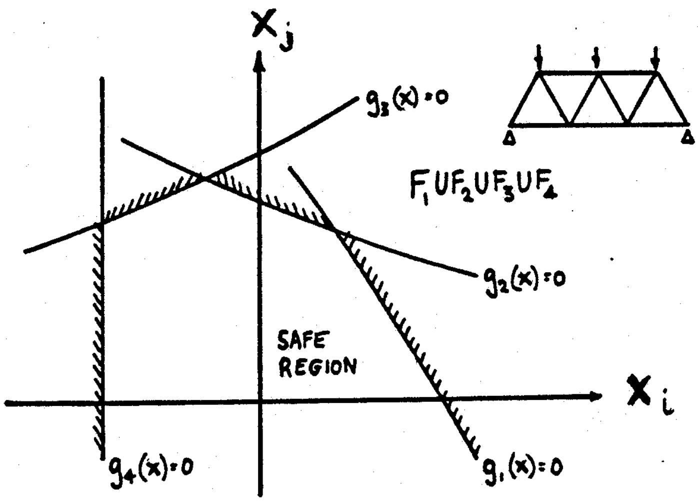

Figure C.6(a): Failure region as union of component failure events for series system

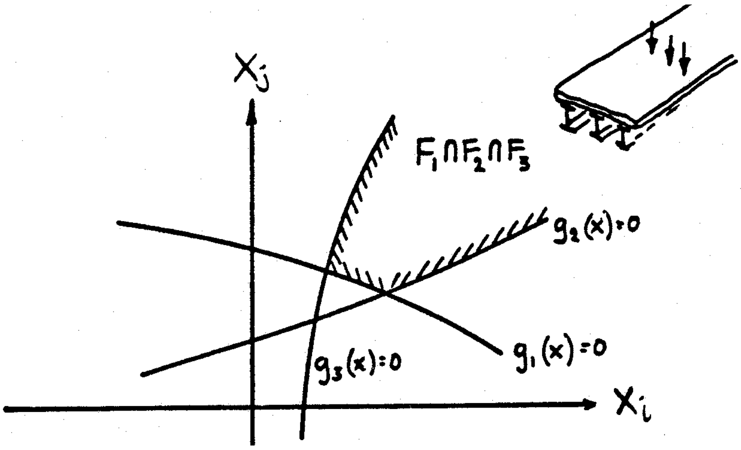

Figure C.6(b): Failure region as intersection of component failure events for parallel system

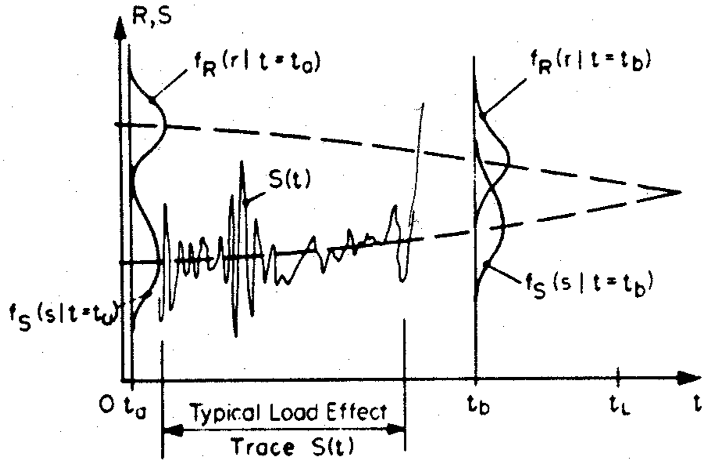

Figure C.7: General time-dependent reliability problem

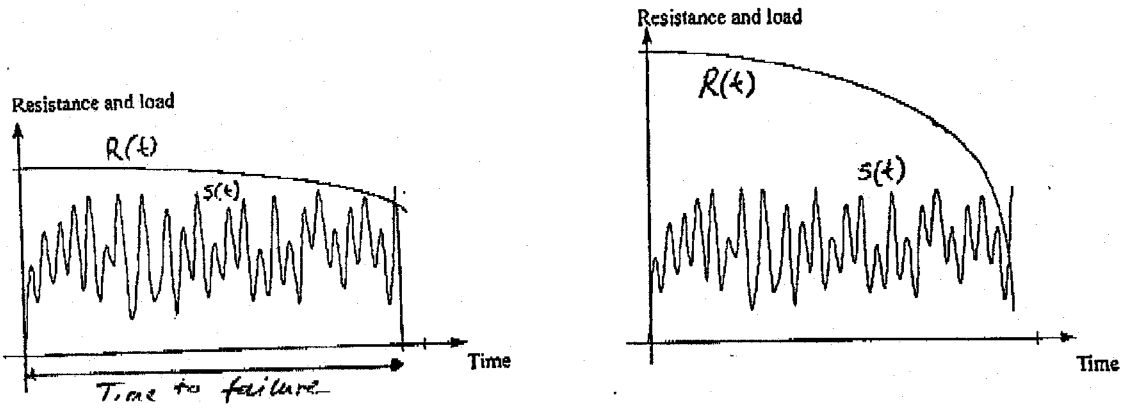

Figure C.8: Schematic representation of crossing problems
(a) slowly varying resistance (b) rapidly varying resistance

Figure C.9: Fundamental barrier crossing problem

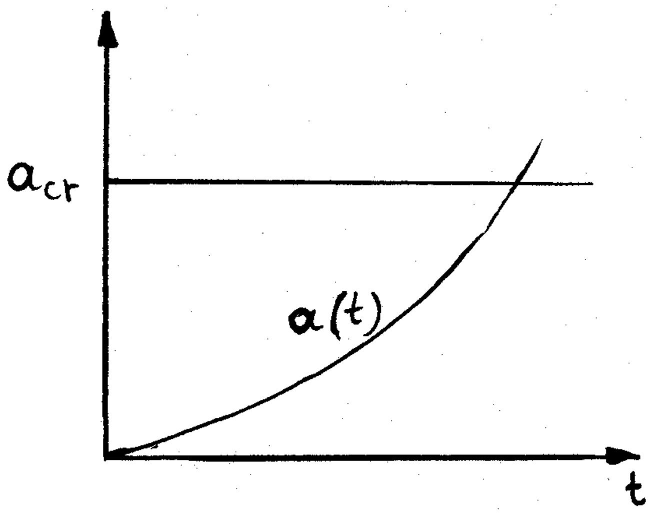

Figure C.10: Damage accumulation problem

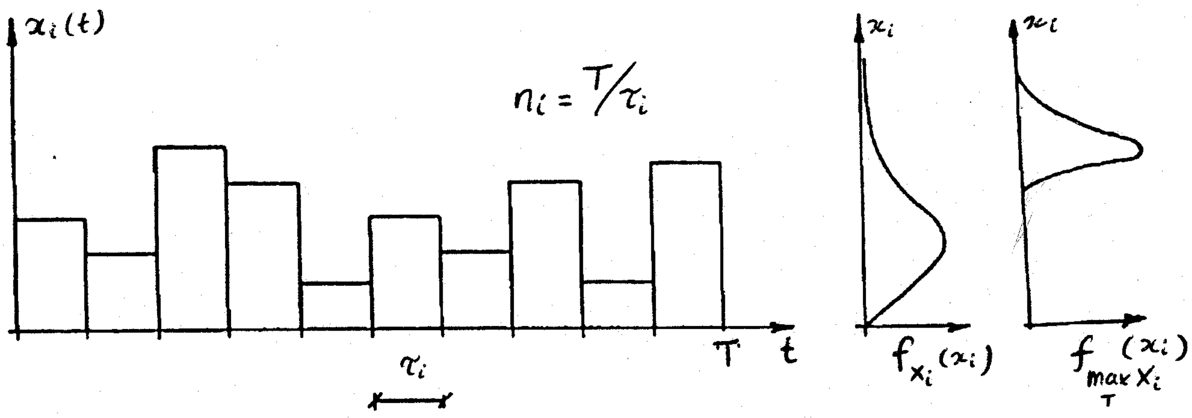

Figure C.11: Realization of an FBC process

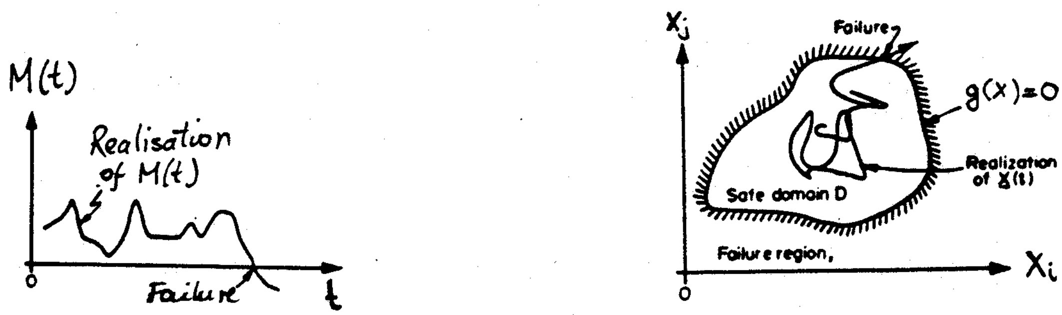

Figure C.12: Time-dependent safety margin and schematic representation of vector outcrossing

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11. Annex D:Bayesian Interpretation of Probabilities

11.1. Introduction

This JCSS Probabilistic Model Code offers distribution functions and corresponding parameter models for loads and structural properties in order to carry out failure probability calculations for comparison with specified reliability targets. This annex gives guidance on the interpretation of both input and results of those calculations.

For the sake of discussion three possible alternatives of interpretation will be mentioned:

1. the frequentistic interpretation
2. a purely formal interpretation
3. the Bayesian interpretation

They will be discussed in the following section.

11.2. Discussion

The *frequentistic* interpretation is quite straight forward. It means that if one observes for a long period of time, say *T*, a large set of say *N* similar structures, all having a failure rate of *p* [1/year], one expects the number of failures not to deviate too far from *pTN*. The deviation should fall within the rules of combinatory probabilistic calculations. Such an interpretation, however, can only be justified in a stationary world where the amount of statistical or theoretical evidence for the all distribution functions is very large. It should be clear that such a frequentistic interpretation of the failure probabilities is out of the question in the field of structural design. In almost all cases the data is too scarce and often only of a very generic nature. Note, however, that a frequentistic interpretation still can be used in a conditional sense. The statement that, given a set of statistical models for the various variables, a structure has some failure probability, is meaningful and helpful.

The interpretation mentioned above given as second, that is the *formal* approach, gives full credit to the fact that the numbers used in the analysis are based on (common) ideas rather than statistical evidence. The probabilistic design is considered as a strictly formal procedure without any meaning or interpretation. Such a procedure can be believed to be a more rich and consistent design procedure compared to for instance a Partial Factor Method or Allowable Stress method. The basic philosophy is that a probabilistic design procedure, running on the average the same design result as its successful predecessors, is a at least as good as or even better then the other methods. So calibration on the average result is the key point and the absolute values of the distributions and the failure probabilities have no meaning at all. An alternative code, prescribing higher standard deviations (resulting in higher failure probabilities) and corresponding higher target probabilities is considered as fully equivalent.

To some extent this formal interpretation has many advantages, but is difficult to maintain. In many cases, it is at least convenient if the various values in the probabilistic calculations have some meaning in the real world. It should be possible, for instance, to consider the distribution functions in this code as the best estimate to describe our lack of knowledge and use them as priors for Bayesian updating procedures in the case of new data. It should also be possible to use the models for decision making in structural design and optimisation procedures for target reliabilities. If this cannot be done the method loses many features of its attraction.

This leads into the direction of a *Bayesian* probability interpretation, where probabilities are considered as the best possible expression of the degree of belief in the occurrence of a certain event. The Bayesian interpretation does not claim that probabilities are direct and unbiased predictors of occurrence frequencies that can be observed in practice. The only claim is that, if the analysis is carried out carefully, the probabilities will be correct if averaged over a large number of decision situations. The requirement to fulfil that claim, of course, is that the purely intuitive part is neither systematically too optimistic nor systematically too pessimistic. The calibration to common practice on the average may be considered as an adequate means to achieve that goal.

The above statement may sound vague and unsatisfactory at first sight. There seems to be an almost unlimited freedom to make unproven assessments based on a very individual intuition only. In this respect, one should keep in mind that:

(1) where data is lacking, statistical parameters like means and standard deviations are not taken as deterministic point estimates but as random quantities usually with a wide scatter; in this code the scatter is not the opinion of an individual engineer, but it is based on the judgement of a group of engineers.

(2) where data is available, estimates can (and often should) be improved on the basis of it; the minimum requirement is that intuitive probability models should not be in contradiction with the data.

Within the Bayesian Probability Theory these starting points have been rigorously formalised. As long as no data is available, a so called uninformative or vague prior estimate is chosen. Given observations, the prior can be updated to a so called posterior distribution, using Bayes' Theorem. For details the reader is referred to Part 3.0, Material Properties, General Principles, Annex A. It should be noted that, in the case of sufficient data, this procedure will tend to probability statements that can be interpreted in a purely frequentistic way.

Data may of course become available in blocks: in such a case the posterior distribution resulting from the first block may be used as a prior distribution for the second data block. That is, in fact, precisely what is present in the various chapters of Parts 2 and 3: the distributions given can often be considered as "data based priors" based on data from a generic world wide population. These models can be "updated" if data of a specific site or a specific producer are available.

Practically spoken, lack of statistical data may lead to (1) uncertainties in statistical parameters (mean, standard deviation, etc) and (2) uncertainty in the type of distribution (normal, lognormal, Weibull, etc). It turns out that the latter type of uncertainty needs unrealistic much data to get a substantial reduction, while calculation results may be very sensitive to it. Also, such large data sets fulfilling the stationarity requirement may hardly be available. It is exactly on this point that there is a need to standardise the input. It should be noted that in this code most distribution types have the nature of a "point estimate", neglecting to some extend the distribution uncertainty.

11.3. Conclusion

The conclusion of the foregoing is that distributions and probabilities in this Model Code should be given a Bayesian degree of belief type interpretation. One may use the distributions as a start for updating in the presence of specific structure related data and as a basis for optimisation.

Some reflections for closure:

(1) The numbers given in this code do not include the effect of gross errors. This is one of the main sources of the deviation between calculated probabilities and failure frequencies in practice.

(2) The justification for a Bayesian probabilistic approach in decision making is that it makes the anyhow inevitable judgement part explicit and minimises the influence of it. The return to so called deterministic procedures because of a lack of statistical data is no realistic alternative.

(3) In the Bayesian procedure the prior, if no explicit data is available, is often referred to as "subjective" or "person dependent". In the case of this code this would not be the right terminology. The priors given are not the result of the ideas and experience of a single individual, but of a large group of experts. This gives the distributions some flavour of objectivity, however, of course, still on an intuitive basis.

(4) The system of given distributions and their use in Bayesian updating and Bayesian decision procedures may be considered as a formal procedure in itself.