Background Documents on Risk Assessment in Engineering

Document #2

Interpretation of Uncertainties and Probabilities in Civil Engineering Decision Analysis

JCSS

Joint Committee of Structural Safety

1 Introduction

Typical engineering problems such as design, assessment, inspection, maintenance planning and decommissioning may be realized to be decision problems subject to a combination of inherent, modeling and statistical uncertainties. This fact was fully appreciated already over 30 years ago and since then the Bayesian decision theory together with Bayesian probabilistic modeling has formed the cornerstones of what is now commonly understood as methods of structural reliability see e.g. Freudenthal et al. (1966), Turkstra (1970) and Ferry Borges and Castanheta (1971).

During the last two decades the area of structural reliability theory has not only matured in a scientific and methodical sense but has also increasingly gained practical significance in a variety of different engineering application fields. These include offshore engineering, aeronautical engineering, naval architecture and civil engineering. As examples of the interdisciplinary success of structural reliability methods should be emphasized reliability based code calibration, reliability based structural (re-)assessment and risk based inspection and maintenance planning. These are all application areas where the methods of structural reliability have proven their value in supporting rational decision making, enhancing the efficiency of risk treatment and at the same time providing a transparent documentation of the basis for decision making.

The introduction of full probabilistic and risk analysis into practice brings, among other things, along the question of the interpretation of probability for a broad class of engineers. And not only engineers. Also politicians, civil servants and citizens will be confronted with safety statements in terms of probability. To some extent, of course, they already do. In the present design process the starting point is a safety level chosen by the politicians. As an example, The Dutch Water Act, approved by the parliament, specifies return periods for water levels and river discharges of for instance 3000 or 10000 years. This already is a matter of confusion for many people and the message that these flood levels or discharges still may occur next year (although with small probability) has to be repeated every time. Despite the general success of structural reliability methods in providing decision support in engineering decision problems still some questions remain regarding the consistent interpretation and modeling of uncertainties and probabilities. Whereas the modeling related difficulties in principle may be solved by improvement of methodology and standardization, the interpretational difficulties are of a more fundamental character and call for careful differentiation between the different decision problems and the information at hand.

The present paper attempts to discuss and put light on some of the interpretational aspects of uncertainty modeling as applied for civil engineering decision support. Following a basic introduction on the interpretation of probability in engineering decision analysis, it is attempted to identify and outline the generic character of different engineering decision problems and to categorize these as prior, posterior and pre-posterior decision problems in accordance with the Bayesian decision theory.

2 UNCERTAINTY – AN ENGINEERING INTERPRETATION

A central role for engineers is to provide a basis for sustainable decision making in regard to the cost efficient safeguarding of personnel, environment and assets in situations where uncertainties are at hand, see e.g. Rackwitz et al (2004) and Faber (2004). A classical

example is the decision problem of choosing the height of a dike. The risk of dike flooding can be reduced by increasing the height of the dike; however, due to the inherent natural variability in the water level a certain probability of dike flooding in a given reference period will always remain. Risk assessment within the theoretical framework of decision analysis can help us in deciding on the optimal dike height by weighing the benefits of reduced dike flooding risks with the costs of increasing the dike height. However, a prerequisite for the risk assessment is that the means for assessing the probability of dike flooding are established, and this in turn requires that a probabilistic model for the future water level is available.

For the purpose of discussing the phenomenon uncertainty in more detail let us initially assume that the universe is deterministic and that our knowledge about the universe is perfect. This implies that it is possible by means of e.g. a set of exact equation systems and known boundary conditions by means of analysis to achieve perfect knowledge about any state, quantity or characteristic which otherwise cannot be directly observed or has yet not taken place. In principle following this line of reasoning the future as well as the past would be known or assessable with certainty. Considering the dike flooding problem it would thus be possible to assess the exact number and heights of floods which would occur in a given reference period (the frequency of floods) for a given dike height and an optimal decision can be achieved by cost benefit analysis.

Whether the universe is deterministic or not is a rather deep philosophical question. Despite the obviously challenging aspects of this question its answer is, however, not a prerequisite for purposes of engineering decision making, the simple reason being that even though the universe would be deterministic our knowledge about it is still in part highly incomplete and/or uncertain.

In engineering decision analysis subject to uncertainties such as Quantitative Risk Analysis (QRA) and Structural Reliability Analysis (SRA) a commonly accepted view angle has developed where uncertainties are interpreted and differentiated in regard to their type and origin, see e.g. Apostolakis (1990). In this way it has become standard also in the Joint Committee on Structural Safety Probabilistic Model Code (2001) to differentiate between uncertainties due to inherent natural variability, model uncertainties and statistical uncertainties. Whereas the first mentioned type of uncertainty is often referred to as inherent or aleatoric (or Type 1) uncertainty, the two latter are refereed to as epistemic (or Type 2) uncertainties.

The aleatoric uncertainty can be associated with experiments like throwing dices or unpredictable phenomena in nature like river discharges, sea levels, wind speeds, quality of concrete, timber strength, soil properties and so on. It is the scatter that one can observe in practice that can be measured and can be described in objective terms. The epistemic uncertainty is related to our lack of knowledge, for instance the limited amount of data in statistical data bases or the fact that our models in general are incomplete or in-accurate. These uncertainties may give rise to great difficulties in interpretation, see e.g. Ferson et al. (2004) for a comprehensive review on the treatment of epistemic uncertainties. Usually it is for instance impossible to find exact methods to describe these uncertainties and one has to fall back on concepts like "intuition", "expert opinion" and "engineering judgment".

The central question is whether the two types of uncertainty can be treated in the same way or that different concepts and procedures should be followed. There is in this respect a direct relationship with the interpretation of probability. The Joint Committee on Structural Safety Probabilistic Model Code (2001) makes a distinction into three possible interpretations:

1.   the frequentist´s interpretation

2.   the formal interpretation

3.   the Bayesian interpretation

The frequentist´s interpretation is quite straight forward and allows only "observable and countable" events to enter the domain of probability theory. Probabilities should be based on a sufficient number of data or on unambiguous theoretical arguments only (as in coin flipping or die throwing games). Such an interpretation, however, can only be justified in a stationary world where the amount of statistical or theoretical evidence is very large. It should be clear that such an interpretation is out of the question in the field of civil engineering applications. In almost all cases the data is too scarce and often only of a very generic nature.

The formal interpretation gives full credit to the fact that the numbers used in a reliability and risk analysis are based on ideas and judgment rather than statistical evidence. Probabilistic design is considered as a strictly formal procedure without any physical interpretation. Such a procedure, nevertheless, is believed to be a richer and more consistent design procedure compared to classical and deterministic design methods.

However, in many cases it is convenient if the values in the probabilistic calculations have some meaning and interpretation in the real world. One example is that one should be able to improve (up-date) the probabilities in the light of new statistical evidence. This leads into the direction of a Bayesian probability interpretation, where probabilities are considered as the best possible expression of the degree of belief in the occurrence of a certain event. The Bayesian interpretation does not claim that probabilities are direct and unbiased predictors of occurrence frequencies that can be observed in practice. The only claim is that the probabilities will be more or less correct if averaged over a large number of decision situations. The requirement to fulfill that claim, is that the purely intuitive part is neither systematically too optimistic nor systematically too pessimistic. Calibration to common practice on the average may be considered as an adequate means to achieve that goal.

It should be clear that a frequentist's approach does not allow a subjective interpretation of probability and, within this framework knowledge uncertainties cannot be treated in the same way as inherent and measurable uncertainties. This does not imply that somebody who favors this view on probabilities fully neglects the other types of uncertainty. He will only use "other ways" to deal with them, as for instance classical tools like confidence bounds, safety factors, safety margins and conservative estimates or additional formalisms like fuzzy set theory Zadeh (1975), belief mass Lair (2000) and so on.

In the Bayesian approach, on the other hand, the two types of uncertainty are treated in the same way. Shortly speaking, the inherent uncertainties are treated in the frequentistic way and the knowledge uncertainties in a degree of belief way. The basic advantage of the Bayesian approach above the other approaches is that the "degree of belief" becomes exactly equal to a "frequentistic probability" in the limiting case of strong evidence like huge statistics or closed theoretical arguments. This property ensures a clear interpretation of the calculations and enables the combination of several sources of evidence. Another advantage is that one has the fully developed and strong theory of probability at ones disposition for both types of uncertainty.

The conclusion of the foregoing is that distributions and probabilities in most civil engineering applications should be given a Bayesian degree of belief type of interpretation.

For further discussion on this point the reader is referred to for instance Benjamin and Cornell (1970), Savage (1954), Lindley (1976), Paté-Cornell (1991, 1996), ISO2394 (1998), Ditlevsen (1988, 1996) and Vick (2002).

In the dike example mentioned before we can imagine that an engineering model might be formulated where future extreme water levels are predicted in terms of a regression of previously observed annual extremes. In this case the uncertainty due to inherent natural variability would be the uncertainty associated with the annual extreme water level, the model uncertainty would be the uncertainty in hydraulic models and the statistical uncertainty would be the uncertainty associated with the statistical distributions and statistical parameters estimated using a limited number of observations of previous annual extremes. The uncertainty associated with the future extreme water level is thus composed as illustrated in Figure 1.
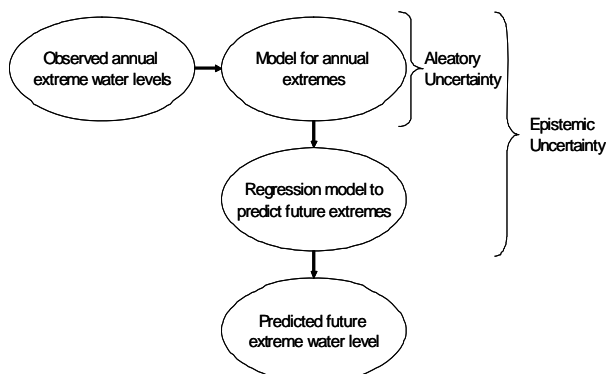

Figure 1.   Illustration of uncertainty composition in a typical engineering problem.

As stated before, once the Bayesian approach has been adopted, there is no need to distinguish between the various sources of uncertainty. According to the Bayesian approach "all uncertainties are equal". However, in practice, like in Orwell's famous book Animal Farm (Orwell, 1945 "some uncertainties are more equal then others". Especially when communicating results of reliability and risk analysis with other people, it is important to point out that the probabilities are based on a mix of objective statistical data and subjective (prior) estimates. The interested parties simply want to know what the subjective choices are and how they may influence the final decision.

Another reason to make a distinction between the different types of uncertainty is that uncertainties that have to do with a lack of knowledge, at least at first sight, might be removed by doing research (model uncertainty) or gathering data (statistical uncertainty) and "aleatoric or inherent uncertainties" cannot. So if the there is a large failure probability and the inherent uncertainties dominate, we go for redesign or re-construction, while in case the epistemic uncertainties dominate we will start investigations. After further reflection, however, this matter is more complex. Some statistical uncertainties cannot be reduced because it is physically impossible to gather data within the time available to take the decision. We cannot gather the wind speeds for the next 1000 years just tomorrow, in order to reduce the statistical (epistemic) uncertainty. And on the other hand, inherent uncertainties like soil or concrete properties of existing structures can be reduced by measurements. Indeed, once any process involving aleatoric uncertainties like the production of concrete or the formation of the ground layers has been completed their type or randomness changes. It may even be a gradual process where uncertainties of the future already may be reduced on the basis of observations now, like for instance in weather predictions. A possible view point on this processes is that the aleatoric variability instantaneously or gradually transforms into a statistical and thus epistemic uncertainty which may be reduced by measurements and

updating procedures. Both interpretations have a certain right of existence and it may depend on the type of problem which one makes most sense. One might even claim a hybrid character of such uncertainties. It should be emphasized, however, that we deal here with a semantic problem in the first place. What is important that some uncertainties can be reduced (may be it is not economical to do so, but that is another point) while others cannot.

Consider again the dike problem. Having formulated a model for the prediction of future extreme water levels and taking into account the various prevailing types of uncertainties the probability of flooding within a given reference period can be assessed and just as in the case of a deterministic and perfectly known universe we can decide on the optimum dike height based on a cost benefit assessment. The type of uncertainty associated state of knowledge, as indicated before, has a time dependency. Following Figure 2 it is possible to observe an uncertain phenomenon when it has occurred. In principle, if the observation is perfect without any errors the knowledge about the phenomenon is perfect. The prediction of the same phenomenon in the future, however, is uncertain as this involves models subject to natural variability, model uncertainty and statistical uncertainty. Often but not always the models available tend to loose their precision rather fast so that phenomena lying just a few days or weeks ahead can be predicted only with significant uncertainty. The near future water levels for a certain dike location depend for instance on present discharges, current rain fall intensities and weather developments. Present measurements may turn the aleatoric random process partly into a prediction with model or statistical uncertainties of a primarily epistemic nature.
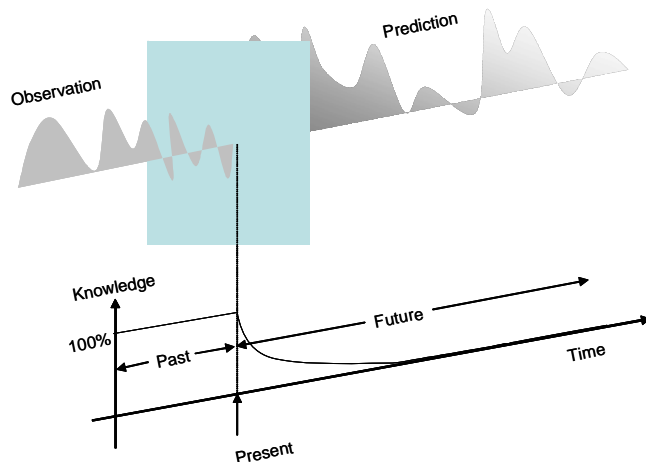

Figure 2.   Illustration of the time dependence of knowledge.

3   UNAMBIGUITY OF PROBABILITIES

The subjectivity in the Bayesian approach leads to the problem that the failure probability of some system may depend on the person or the group that is making the design. If another group of experts is asked, they may come up with different estimates. Also when research is going on and new information becomes available, the safety estimate of the considered system may change. For many people this is a difficult point to swallow. People like to see the reliability of a dike as an unambiguous property of the dike. The subjectivity is destroying this ideal picture. However, it is worth to realise that the element of unambiguity is also

present in the world of the objective probabilities. Also in an objective analysis without any intuition or expert opinion, one may arrive at different probabilities for the same event. This point may be illustrated by the following examples:

3.1   Car example

As an example from the daily practice, consider a car with two types of braking systems, 10 percent of the cars have type A and 90 percent have type B installed. Unfortunately the brakes of type A prove to be unreliable, but it is not known in which car which type of brake is present. Some accidents already have happened. So the car industry invites all cars to come to the garage for a repair.

An arbitrary owner of the car type has driven without any concern until the message comes that something might be wrong. After that he will drive only very carefully and preferably to the closest garage. If one finds there that brakes of type A are present, the drivers' carefulness was justified: his car was unsafe indeed. It will be repaired and he will drive safe again. However, if one finds type B brakes, the car was considered as unreliable without justification (from the posterior point of view) and it can continue its journey safely without anything done. Now notice that the car of type B never changed intrinsically: neither when the message came, nor when it was inspected. The reliability of the car as "a car property" never changed. But nevertheless its reliability to the owner changed. It is very important to understand this distinction. The point is that usually we cannot base our decisions on the "real reliability" of the car, because it often is unknown. We have to base our decisions on our knowledge of the car. The lack of knowledge is sometimes the only thing that matters and cannot be neglected.

It may also be illustrative to give some numbers for this example: Let the failure probability of the brakes type A be 10^{-6} per km and for type B 10^{-9} per km, then the failure probability for the driver of an arbitrary car is given by:

$$p = 0.1 \cdot 10^{-6} + 0.9 \cdot 10^{-9} = 0.1009 \cdot 10^{-6} \approx 10^{-7} \text{ per km}$$

The intellectually difficult point for most people will be that there simply is no car with this probability: there exists only cars of 10^{-6} and 10^{-9}, but none of 10^{-7}.

3.2   Ground example

Consider an ideal ground structure (e.g. a foundation or embankment) where all calculation models are without dispute and all statistical models are well known. Let, in this model, the friction angle φ be a spatially varying stochastic field with the following properties.

-       the field is described by the mean value μ(φ), the standard deviation σ(φ) = 4° and the correlation distance d(φ) = 100 m;

-       for the mean μ(φ) one has found in comparable layers a value of 30° on the average with a scatter 10 percent.

We may now proceed in three different directions:

1) Consider μ(φ) as a random variable, having a mean of 30o and a standard deviation of 3o.
2) Do a local soil mechanics survey resulting in a deterministically known value for μ(φ)

3) Do a very detailed local soil mechanics survey leading to a deterministically known value for φ at every point of the layer and no uncertainty is left at all.

All three situations lead to a certain value of the failure probability. In most cases we may expect Pf 1 > Pf 2 > Pf 3 , but not always. Nevertheless, all three probabilities may be considered as "objective and real". The differences concern the amount of available information and the corresponding definitions of the populations. Most people would be inclined to consider the uncertainties as in all three cases as aleatoric. The fluctuations in the ground properties are the result of a random natural process. There is no model uncertainty, and for every chosen population there is no lack of data. Referring to the previous discussion on this point, however, we could also maintain that the uncertainty is of the epistemic type. The uncertainty is purely the result of our lack of knowledge on the details pf a more advanced level of schematisation. Doing measurements may solve this lack of knowledge. Again we should conclude that the border between aleatoric and epistemic uncertainties is not sharp and of a primarily semantic type. In this respect it should also be pointed out that even without explicit research, the failure probability may change. For instance if the dike has survived a number of load situations without failure, its reliability has increased. How much depends on the variability of the load and the resistance.

4   PRESENTATION OF EPISTEMIC UNCERTAINTIES

Consider the following safety margin Z:

$$Z = R - S - E \tag{1}$$

Where R is the resistance part with "aleatoric uncertainties", S is the load part with "aleatoric uncertainties" and E is the part that represents the "epistemological uncertainties", for instance the statistical uncertainties in the loads or the model uncertainty in the resistance. The values of means and standard deviations have been summarized in the following table:

Var	type	μ	σ	Description
R	normal	14	2.0	Resistance
S	normal	5	1.0	Load
E	normal	0.0	2.0	Lack of Information

The reliability index may then be calculates as $\beta = 3$ corresponding to a failure probability of about 10^{-3}; however, in this analysis the epistemological uncertainties have been treated exactly in the same way as the inherent uncertainties in R and S. A possible indication for the importance of the epistemological uncertainties, of course, is the influence coefficient α as calculated in standard FORM. As an alternative and visually attractive alternative one might present β as a distribution indicated in Figure 1. The curve can be found by calculating the reliability index, conditional upon the variable E:

$$\beta = \Phi^{-1}(P(Z \leq 0 | E)) \tag{2}$$

and using this relation for the transformation from E to β. In Equation (2) $\Phi(\cdot)$ is the standard normal distribution function. Figure 3 shows that the reliability index β would be equal to 4.02 if all epistemological uncertainties would be absent (this is easy to verify).
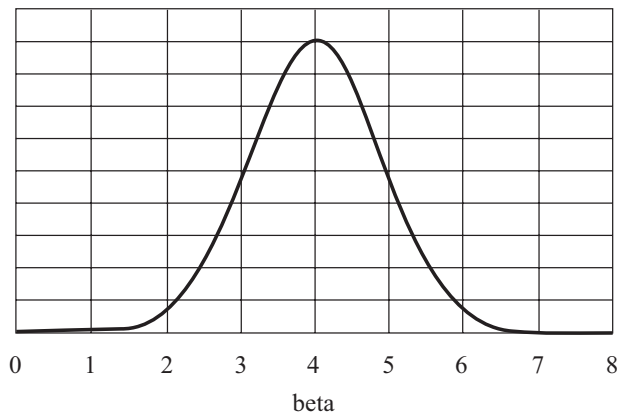

Figure 3.   The reliability index   presented as a random variable; the randomness is due to the epistemological uncertainties.

The scatter indicates "how well we know" this index. A narrow band indicates a small influence of the subjective influences, a wide band the contrary. The standard deviation of the curve may be proven to be equal to $\alpha E/\sqrt{(1-\alpha E^2)}$, where αE is the FORM influence coefficient for the variable E.

The reason that the two types of uncertainty are not considered as fully equivalent is, as said before, that one type is "real" and the other type seems to be present "in the mind of the engineer" only. The "taste" is different and so people want to know what is the one and what is the other. Uncertainty in the mind of the engineer looks less harmless. Maybe the situation is not "really dangerous" after all. Some people would even tend to forget it at all. This of course is not acceptable. It was already mentioned that no designer should ever forget about these uncertainties. If not allowed or accustomed to presenting them by way of probabilities, they will use other methods as indicated in section 2.

Separate presentation gives a feeling of the sensitivity to assumptions and estimates. It is interesting to observe that if one combines the both types of uncertainty, the question to the sensitivity of the assumptions and estimates pops up again. It is true that one will get other answers if different input values are used. However, this may never happen in such a way that a change of say a mean value into another "equally likely value" results in a major difference in final outcome. If that is the case, the uncertainty was not quantified in a proper way. As a rule of thumb one could say that if a shift of about a standard deviation is considered as a shift to an equally possible value, the standard deviation in the analysis was too small.

5 EPISTEMIC UNCERTAINTIES AND EXPERT OPINIONS

Within the Bayesian framework lack of data may be replaced by engineering judgement and expert opinions. We may ask an expert to estimate the probability that a certain event will happen or we may ask his best guess for a certain variable. Additionally we may ask him to express his uncertainty with respect to the number given. In the case of a random variable we may also ask directly for the whole distribution. Essentially, however, this comes down to ask for a mean, a standard deviation and/or a set of other parameters. So it is not necessary to discuss this separately.

In some cases we have only a single expert and in other cases a group. In the last case, which by the way is to be preferred, we have to combine the results. Let us start the discussion with the simple case that we ask two experts for the probability of event A occurring. Suppose we get the answers:

Expert 1: P(A) = 0.8

Expert 2: P(A) = 0.6

The most obvious thing now to do now is to conclude is that we will use 0.7 in our analysis. But is this always correct? It turns out that the details of the background are important. Consider first the following case: expert 1 looks for typical characteristics in the problem at hand that may lead to the occurrence of A or not. Let it be his experience that he is correct in 80 percent of the cases. A similar mechanism may be present behind the reasoning of expert 2. If their indicators are independent and if there is no prior preference regarding A or $\overline{A}$ (that is $P(A) = P(\overline{A}) = 0.5$ if no expert is consulted), one may derive on the basis of Bayes' Theorem:

$$P(A|A_1, A_2) = cP(A_1 \cap A_2|A)P(A)$$
$$= c \cdot 0.6 \cdot 0.8 \cdot 0.5 = 0.86$$

$$P(A|A_1, A_2) = cP(A_1 \cap A_2|\overline{A})P(\overline{A})$$
$$= c \cdot 0.4 \cdot 0.2 \cdot 0.5 = 0.14$$

Here A_1 indicates that expert 1 "expects A" and A_2 that expert 2 "expects A"; c is a normalising constant. The result is that adding expert 2 increases the probability that A will occur, rather than that it decreases. This makes sense: we have already the opinion of expert 1 that A most probably will occur and we find another expert who more or less supports his opinion. This should increase the probability that A will occur. It is only the non-expert, who can only predict A with a probability of 0.5, that will not increase the estimate by expert 1.

However, this is not the only way the experts may have arrived at their opinion. In another situation, maybe, both experts had just observed a small number of observations, say expert 1 had seen A occurring four times out of five similar situations and expert B had seen it happen only 3 times. If those observations were not overlapping one might in that case indeed be better of with the first approximation of simply averaging the estimates.

Of course one might say that these are not purely expert opinions as in both cases some elementary observations and data are being used. On the other hand, there is always some kind of reasoning behind the estimate of the expert. It is absolutely essential to get this reasoning on the table as it may help to construct a combined judgement.

In a similar way we may ask a number of experts the value of x representing some deterministic physical entity. Let us ask them for their best guess and let us also ask them to quantify their uncertainty. Assuming normal distributions we then might have:

$$f(x) = \frac{1}{n} \sum_{i=1}^{n} \varphi(\frac{x - \mu_i}{\sigma_i}) \tag{3}$$

This seems a very straightforward approach, but again we have other options. First let us consider a slightly different case where we use a block distributions in stead of normal distributions. In this case the result for instance of two experts could be:

Expert 1: X is in the interval [10, 13]

Expert 2: X is in the interval [11, 15]

Following the procedure from above we would get the distribution indicated in Figure 4, upper diagram. However, following elementary rules from formal logic we should conclude that x must lie with 100 percent probability in the interval [11,13], resulting in Figure 4, lower diagram. Of course, this requires that we trust the experts completely, and this usually is exactly the bottleneck of experts opinions.
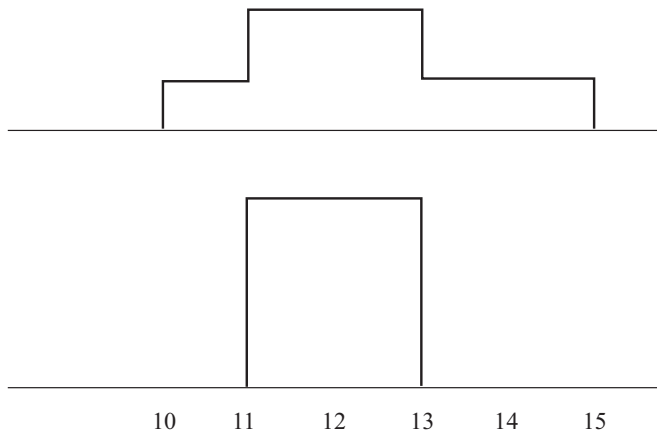

Figure 4.   Two possible options to combine expert opinion 1 (X is uniform in [10,13]) and expert opinion 2 (X is uniform in [11,15]).

The equivalent of this second type of reasoning for normal distributions could be reached by considering the experts opinions to be data of x_i ,, where every measurement is associated with uncertainty. We use Bayesian updating formula. There are two approaches:

 (1) We consider the uncertainties defined by the experts as "calibrated measurement uncertainties". We then have:

$$f''(x) = c \prod_{i=1}^{n} \varphi(\frac{x_i - x}{\sigma_i}) f(x)$$
(4)

Where $f(x)$ is a prior estimate, x_i and σ_i are the best estimate and uncertainty from expert i ($i = 1,..,n$) and $f''(x)$ is the updated distribution for the unknown quantity x. This approach is proposed Mosleh and Apostolakis (1982, 1986).

(2) We treat the best guesses of the experts just as data points in an experiment to find x. The measurement error equals the scatter in the estimates, enlarged by the uncertainty expressed by each individual expert. In that case, using Bayesian analysis, we find:

$$f(x) = \iint \varphi(\frac{x - \mu_X}{\sigma_X}) f''(\mu_X, \sigma_X) d\mu_X d\sigma_X$$
(5)

where $f''(\mu_X, \sigma_X) = c \prod_{i=1}^{n} \varphi(\frac{x - \mu_X}{\sqrt{\sigma_X^2 + \sigma_i^2}}) f(\mu_X, \sigma_X)$

For this estimate one also needs a prior. We may refine the procedure by giving every expert not just one data point, but by letting his estimate count for a number of say 3 or 10 data points. Especially in the case of one expert this is necessary in order to avoid extremely large uncertainties.

Figure 5 shows a number of results for the case of two experts. We consider the situation that the two experts have different views. One expert thinks that $x = 10$ and the other one that $x = 14$. Both give an uncertainty of $\sigma = 1$. Which is the alternative to be chosen? Equation (4) is attractive and is the equivalent from our "formal logic approach" in Figure 4, lower diagram. But which engineer is inclined to put so much faith in his experts that he really dares to make the choice. Equations (3) and even better (5) seem to be more acceptable from this point of view. Different views by experts is a clear indication that things have not been settled and one should not be too optimistic. In addition one should keep in mind that experts often tend to be overconfident in expressing the accuracy of their own estimates (e.g. De Wit, 2001).

Until now we have treated all experts to be equally skilful. An interesting option is to give the experts a number of questions for which they do not know the answer, but the interrogators do. We then may adjust the weights of the experts on the basis of their score on these "seed questions". We will not go into this matter in this paper. The reader is referred to Cooke (1991).

In order to get some impression of the reliability of the various models used in the Dutch safety project for the flood protection systems, a number of experts have been interrogated in order to get a grip on the accuracy of the various models used (Cooke et al, 1998). In this particular project the experts were asked to give a 0.50 fractile as best estimate and express their uncertainty by stating the 0.05 and 0.95 fractiles.

The distributions obtained this way have been added by giving a weight to every expert. One procedure was to give equal weights and the second one was to give weights based on how the experts scored
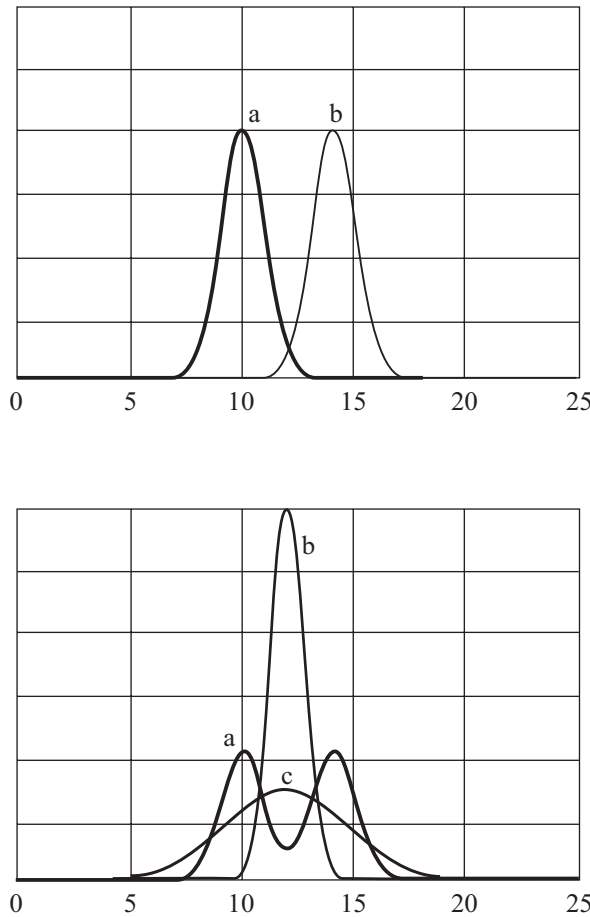
Figure 5.   upper: Two expert opinions: (a) expert 1 X=10 with uncertainty σ=1; (b) expert 2 X =14 with σ=1;
lower: Combination of expert opinions according to (a) equation (3), (b), equation (4) and (c) equation (5)

on a number of questions for which the interviewers but not the experts knew the expert answer. There were two criteria: the answers should be within the margins of the expert and the margins should not be too narrow, otherwise the expert would be of no help. Details of the analysis can be found in Cooke (1991) and Cooke (1998). Some results are, for the case of illustration, presented in Table 2.

Table 2.   Examples of model uncertainties found by expert elicitation in The Netherlands)

	mean	scatter
local water levels along rivers	0.0	σ=0.15m
wave development due to wind	1.0	V=0.15
wave run up on the dike slope	1.0	V=0.50
critical crest discharge	1.0	V=0.50

6   ENGINEERING DECISION ANALYSIS

Without giving the theoretical argumentation here (see e.g. Von Neumann and Morgenstern (1943) and Ditlevsen and Madsen (1996)), it is simply stated that a fundamental principle in decision theory is that optimal decisions must be identified as those resulting in the highest expected utility.

In typical engineering applications the term utility may directly be translated into consequences in terms of benefit, costs, fatalities, environmental impact etc. In these cases the optimal decisions are those resulting in the highest expected benefit, lowest expected costs, lowest expected number of fatalities and so on. Moreover, if costs and fatalities and/or other attributes form part of the decision problem, full consistency may only be ensured if these attributes are expressed in terms of a common utility. This has for a long time been considered to represent a controversial problem, but resent work by Nathwani et al. (1997) and Rackwitz (2001) and emphasizes the need to do so and also provides the required philosophical and theoretical framework. The weighting of the attributes has to be done somehow, explicitly or implicitly, in order to make a decision. Thus, in order for the decision maker to be sure that the decision is made in accordance with her/his preferences, the weighting should be made in a transparent way.

As the immediate consequence of the fact that any activity planned or performed in order to reduce and/or control the risk is only directly quantifiable in terms of costs, the most straightforward approach is to associate utility with monetary consequences. However, in some cases the requirements given by legislation are formulated in terms of number of fatalities and in such cases it is necessary to assess the risk both in terms of expected benefit or costs and in terms of the expected number of fatalities.

Table 3.   Categorization of engineering problems as decision problems.

Engineering Problem	Decision Theoretical Problem		
	Prior	**Posterior**	**Pre-posterior**
Risk assessment for verification Design and strengthening optimization	X		
Calibration of - risk acceptance criteria - code formats (γ, ψ)	X		
Reliability updating for - service life extensions - re-qualification		X	
Planning of collection of information - tests - experiments - proof load levels			X
Inspection and maintenance planning			X

Consistent decision making subject to uncertainty is treated in detail in Raiffa and Schlaifer (1961) and Benjamin and Cornell (1970). In the following an introduction to three different decision analyses is given, namely prior, posterior and pre-posterior decision analysis. Other

aspects on decision analysis in engineering applications are treated in Apostolakis (1990), Paté-Cornell (1996) and Faber and Stewart (2003).

6.1 Prior Decision Analysis

The simplest form of the decision analysis is the so-called prior-analysis. In the prior-analysis the risk (expected utility) is evaluated on the basis of statistical information and probabilistic modeling available prior to any decision and/or activity. This prior decision analysis is illustrated by a simple decision tree in Figure 6. In prior decision analysis the risk (expected utility) for each possible decision activity/option is evaluated in the principal form as

$$R = E[U] = \sum_{i=1}^{n} P_i \cdot C_i \qquad (6)$$

where R is the risk, U the utility, P_i is the i'th branching probability (the probability of state i) and C_i the consequence of the event of branch i see Figure 3.

Prior decision analysis in fact corresponds closely to the assessment of the risk associated with an activity. Prior decision analysis thus forms the basis for the simple comparison of risks associated with different activities. The result of a prior decision analysis might be that the risks are not acceptable and the risk reducing measures needs to be considered.

In structural engineering a typical prior decision analysis is the design problem. A design has to be identified which complies with given requirements to the structural reliability. The representation of uncertainties is made on the basis of the existing information about materials and loads, however, as these have not occurred yet the probabilistic modeling involve both aleatoric and epistemic uncertainties.
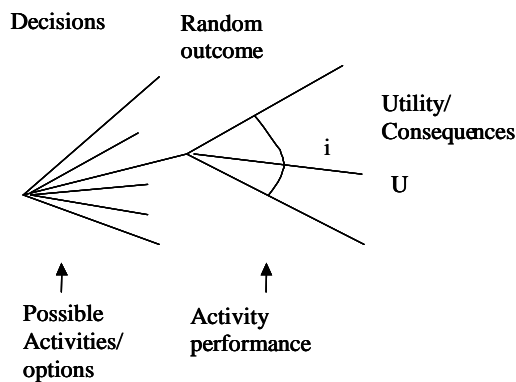

Figure 6.   Decision tree for prior and posterior decision analysis.

6.2 Posterior Decision Analysis

Posterior decision analysis is in principle of the same form as the prior decision analysis, however, changes in the branching probabilities and/or the consequences in the decision tree reflect that the considered problem has been changed as an effect of e.g. risk reducing measures, risk mitigating measures and/or collection of additional information. Posterior decision analysis may thus be used to evaluate the efficiency of risk reducing activities with known performances. The posterior decision analysis is maybe the most important in engineering applications as it provides a means for the utilisation of new information in the decision analysis – updating - which will be described in short in the following. The reader is referred to e.g. JCSS (2000) for a guideline on reliability updating for assessment of existing

structures or Faber (2001) for a more general overview of reliability updating in civil engineering.

6.2.1 Uncertainty updating – updating of random variables

Inspection or test results relating directly to realizations of random variables may be used in the updating. The distribution parameters are initially (and prior to any update) modeled by prior distribution functions.

By application of Bayes theorem, see e.g. Lindley (1976), the prior distribution functions, assessed by any mixture of frequentistic and subjective information, are updated and transformed into posterior distribution functions.

Assume that a random variable X has the probability distribution function $F_X(x)$ and density function $f_X(x)$. Furthermore assume that one or more of the distribution parameters, e.g. the mean value and standard deviation of X are uncertain themselves with probability density function $f_Q(q)$. Then the probability distribution function for Q may be updated on the basis of observations of X, i.e. $\hat{x}$.

The general scheme for the updating is

$$f_Q''(q/\hat{x}) = \frac{f_Q'(q)\,L(q/\hat{x})}{\int_{-\infty}^{\infty} f_Q'(q)\,L(q/\hat{x})\,dq} \tag{7}$$

where $f_Q(q)$ is the distribution function for the uncertain parameters Q and $L(q|\hat{x})$ is the likelihood of the observations or the test results contained in $\hat{x}$. Here ´´ denotes the posterior and ´ the prior probability density functions of Q. The observations $\hat{x}$ may not only be used to update the distribution of the uncertain parameters Q, but also to update the probability distribution of X. The updated probability distribution function for X $f_X^U(x)$ is often called the predictive distribution or the Bayes distribution. The predictive distribution may be assessed through

$$f_X^U(x) = \int_{-\infty}^{\infty} f_X(x|q)\,f_Q''(q/\hat{x})\,dq \tag{8}$$

In Raiffa and Schlaifer (1961) and Aitchison and Dunsmore (1975) a number of closed form solutions to the posterior and the predictive distributions can be found (also collected in JCSS (2001)) for special types of probability distribution functions known as the natural conjugate distributions. These solutions are useful in updating of random variables and cover a number of distribution types of importance for reliability based structural reassessment. However, in practical situations there will always be cases where no analytical solution is available. In these cases FORM/SORM techniques (Madsen et al. (1986)) may be used to integrate over the possible outcomes of the uncertain distribution parameters and in this way allow for assessing the predictive distribution.

6.2.2 Probability updating - updating of uncertain relations

In many practical problems the observations made of realizations of uncertain phenomena cannot be directly related to random variables. In such cases other approaches must be followed to utilize the available information. Given an inspection result or otherwise an observation of an outcome of a functional relationship between several basic variables, probabilities may be updated using the definition of conditional probability or its extension known as Bayes formula:

$$P(F|I) = \frac{P(F \cap I)}{P(I)} = \frac{P(I|F)P(F)}{P(I)} \tag{9}$$

F = Failure

I = Inspection result

For a further evaluation of Equation (4) it is important to distinguish between the types of inspection results. The inequality type information $h(X) < 0$ may be elaborated in a straightforward way. Let F be represented by $M(\mathbf{x}) \leq 0$, where M denotes the safety margin. We then have:

$$P(F|I) = \frac{P(M(\mathbf{X}) \leq 0 \cap h(\mathbf{X}) < 0)}{P(h(\mathbf{X}) < 0)} \tag{10}$$

where $\mathbf{X}$ = vector of random variables having the prior distribution $f_{\mathbf{X}}(\mathbf{x})$. This procedure can easily be extended to complex failure modes and to a set of inspection results ($\cap h_i(x) < 0$).

6.3 Pre-posterior Decision Analysis

Pre-posterior decision analysis may be illustrated by the decision tree shown in Figure 4. Using pre-posterior decision analysis optimal decisions in regard to activities, which may be performed in the future, e.g. the planning of risk reducing activities and/or collection of information may be identified. An important pre-requisite for pre-posterior decision analysis is that decision rules need to be formulated specifying the future actions, which will be taken on the basis of the results of the planned risk reducing activities.
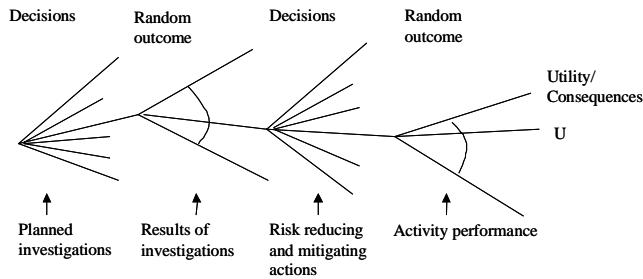

Figure 7.   Decision tree for pre-posterior decision analysis.

In pre-posterior decision analysis the risk (expected utility) for each of the possible risk reducing activities is evaluated as

$$R = E[U] =$$

$$\min_a E'_Z \left[E''_{|Z} [U(a(z), z)] \right] = \min_a E'_Z \left[\sum_{i=1}^n P''_i(a(z), z) C_i(a(z)) \right] \tag{11}$$

where $a(z)$ are the decision rules describing the different possible actions that can be take on the basis of the result of the considered investigation z, $E[\ ]$ is the expected value operator. ´ and ´´ refer to the probabilistic description of the events of relevance based on prior and posterior information respectively see e.g. Lindley (1976).

Pre-posterior decision analysis forms a strong decision support tool and has been intensively used for the purpose of risk based inspection planning see e.g. Faber (2002). However, so far pre-posterior decision analysis has been grossly overlooked in risk assessments in general.

7   UNCERTAINTY REPRESENTATION

As mentioned in the foregoing it is, for several reasons, important to differentiate between the different types of uncertainty in the probabilistic modeling of uncertain phenomena. We will distinguish between the various decision situations

7.1   *Uncertainty modeling in prior decision problems*

For prior decision problems the classical terminology seems to be the most appropriate: all scatter from a natural source is called aleatoric and model and statistical uncertainties are epistemic. The subdivision actually is relevant only in the communication with the outside world. People usually accept easily the natural scatter as a fact of life but want to know explicitly the details and effects of the more subjective parts of the reliability models.

7.2   *Uncertainty modeling in posterior decision problems*

In engineering decision analysis posterior decision problems typically involve the updating of the probability of a future adverse event F, P_F^U conditional on the observation of an event I which can be related to the adverse event. Such observations may in general be considered as being indications about the adverse event. The probability P_F^U may be assessed by Equation (4)

$$P_F^U = P(F|I) = \frac{P(F \cap I)}{P(I)} \tag{12}$$

Taking basis in Equation (7) a simple case is now considered where the adverse event is a future ($\tau \in [t, T]$) failure event in terms of a load $S(\tau)$ exceeding the resistance R of an existing structural component. Furthermore it is assumed that the indicator I is the event that the component has survived all previous realizations of the loading $S(\tau)$ $\tau \in [0, t]$. Then we may write Equation (7) as

$$P_f^U(T) = P(F|I) =$$
$$\frac{P\left(\left\{\min_{\tau \in [t,T]}(R - S(\tau)) \le 0\right\} \le 0 \cap \left\{\min_{\tau \in [0,t]}(R - S(\tau)) > 0\right\}\right)}{P\left(\left\{\min_{\tau \in [0,t]}(R - S(\tau)) > 0\right\}\right)} \tag{13}$$

In accordance with the considerations made in the previous R is an epistemic uncertainty since it has already had its realization but it is still unknown and thus uncertain. As long as no other information is available it would be consistent to model the epistemic uncertainty associated with R using the same model assumptions (distribution type etc.) as before R had its realization. $S(\tau)$ is "in principle" an aleatoric uncertainty (assuming that no model and/or statistical uncertainties are involved in the modeling of the load) when we consider future loads i.e. for $\tau \in [t, T]$. The uncertainty associated with $S(\tau)$ is of an epistemic nature when we consider already occurred load events, i.e. $\tau \in [0, t]$. The wording "in principle" is used because the temporal dependency characteristics of the loading $S(\tau)$ play a significant role.

If the load events (or extreme loads) in consecutive time intervals are assumed or at least conditional independent – a relatively normal case in engineering problems – then the consideration outlined in the above are valid. This also implies that the uncertainty associated with the future loading cannot be updated on the basis of observations of the past loading. However, if the load events in consecutive time intervals are dependent then a part of the uncertainty associated with the future loading becomes epistemic as soon as its first realization has occurred. The "size" of the part depends on the temporal dependency.
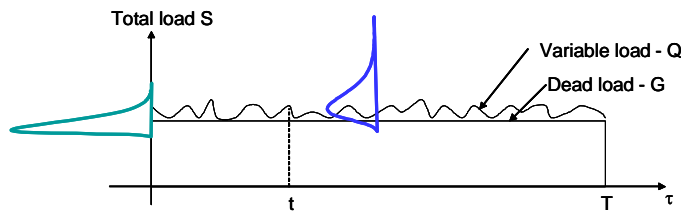
Figure 8.   Illustration of a load with high degree of temporal dependency.

In Figure 5 it is illustrated how the load events in consecutive time intervals may be highly dependent due to e.g. a dominating dead load component. Before the dead load component is realized the loading in the future might be subject to aleatoric uncertainty only. As soon as the dead load component is realized a large part of the uncertainty associated with the future loading becomes epistemic. This effectively implies that this part of the uncertainty associated with the future loading can be updated on the basis of observations of the past loading. In other words – the part that can be updated is exactly the epistemic part of the uncertainty. If the probabilistic modeling of the uncertainties and the probability updating is performed in accordance with Equation (8) and the considerations outlined in the above then, the resulting probabilistic modeling and the representation of the different types of uncertainties is consistent. However, if in the representation of the adverse event and the updating event the different types of uncertainty and the temporal dependency is not consistently taken into account, the results may become grossly erroneous and non-physical.

Other posterior decision problems could be considered such as e.g. reliability updating of fatigue sensitive welded connections using observations of fatigue crack growth. However, it may be realized that the considerations outlined in the previous are valid for this class of problems also.

7.3   Uncertainty modeling in pre-posterior decision problems

As can be realized from Equation (6) pre-posterior decision problems may be seen as a series of posterior decision problems for which the optimal solutions are averaged out over the all prior uncertainty. The formulation of each of the posterior decision problems is based on an updated probabilistic model of the prevailing uncertainties assuming a given "outcome of nature". Therefore the considerations made for posterior decision analysis, concerning the treatment of uncertainties are also valid for pre-posterior decision problems.

8   DISCUSSION AND CONCLUSIONS

The present paper initially gives an engineering interpretation of uncertainty with the purpose of providing a basis for the evaluation of the consistency and appropriateness of the probabilistic modeling as applied in engineering decision analysis. Thereafter a summary presentation of the prior, the posterior and the pre-posterior decision analysis is provided together with a categorization of typical engineering decision problems into different types of decision analysis.

Finally an outline of the consistent treatment of uncertainties in probability updating problems as encountered in prior, posterior and pre-posterior decision analysis is given.

Even though in many cases it is not absolutely necessary to consider the detailed characteristics of the uncertainties prevailing engineering decision problems it is always useful and instructive to think about the nature of the various types of uncertainty. It is relevant in the discussion with the outside world but also in relation to updating. When

discussing results of reliability and risk analysis with the outside world it is important to distinguish primarily between the objective probabilities related to scatter and uncertainty from a natural origin on one hand and subjective probability estimates for knowledge (epistemic) uncertainties. When considering updating it is important to realize that how uncertainties change characteristics as function of both the point in time where they are looked upon and as function of the "scale" of the modeling used to represent them. To some extent, all uncertainties that can be reduced, regardless their origin, are of an epistemic nature. Note however, that not all epistemic uncertainties can be reduced, as for instance statistical uncertainties for which we need observation of the future.

Understanding is a prerequisite for the consistent treatment. As an example - choosing a complicated model with many parameters for the description of uncertain phenomena may result in predictions with significant uncertainties due to epistemic uncertainties as opposed to more simple models dominated by aleatory uncertainties. However, the model dominated by epistemic uncertainties has the potential for reducing the uncertainties by updating.

9 ACKNOWLEDGEMENTS

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