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The Philosophy Behind the Life Quality Index and Empirical Verification

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1. Preface

The following memorandum collects and discusses most of the relevant material presently available for setting up rational risk acceptance criteria and risk reduction expenses. They are based on the so-called life quality index and utility considerations. The paper attempts to support all basic assumptions by empirical findings. Much of the material presented herein is not available elsewhere. The memorandum is meant to be the basis for practical decisions.

Before starting we try to estimate the size of risk reductions we are going to discuss. Overall crude mortality (per year) is about 0.01 in industrial countries but only 3 in 10000 are not due to natural causes. If one subtracts from this number those deaths which are induced by voluntary risky activities (sports and some traffic accidents) and those which are unavoidable such as house accidents, climbing stairs, etc., then, the reduction of a mortality of about 0.0002 or less is the subject of our study. It may be a little larger because certain risks typical for industrialized countries like air pollution are not separated out in the usual statistical records.

Also, we estimate the result of a (crude) mortality reduction $dm/year$ in terms of increases in life expectancy $d\ell$ using a European life table (the corresponding theory is developed later).

dm/a	$d\ell$
10^{-7}	4 hours
10^{-6}	1.5 days
10^{-5}	2 weeks
10^{-4}	5 month
10^{-3}	4 years

It is concluded that we are discussing small changes in mortality with changes in life expectancy between several days to a few years.

2. Derivations and Empirical Verification of the Life Quality Index

The following considerations are valid only for public reduction of involuntary risks of an anonymous member of society. If we wish to recommend rational choices monetary valuation of human

life cannot be avoided. Attributes like life expectancy, age, income and consumption must be incorporated. In particular, let $g > 0$ be the fraction of the real (inflation-free) GDP available for risk reduction and $\ell > 0$ the life expectancy at birth. A factor adjusting ℓ for life years in good health can be included if necessary [40]. In order to clarify the essentials of the life quality index all steps of derivation are presented. Long life and consumption (wealth) are, in fact, among the primary concerns of humans in a modern society as shown by Cantril [15] and many similar, more recent empirical studies.

Assume that the life quality index L , a suitable composite social indicator, is the product of a function of g (as a measure of the quality of life) and another function of the time $t = (1 - w)\ell$ to enjoy life (as a measure of the quantity of life) where w ($0 < w < 1$) is the time (as a fraction of life expectancy) to be spent in (paid) work. Thus, the new variable w is also considered as an indicator of life quality. The individual can now increase leisure time by either increasing life expectancy by risk reduction or by reducing the time spent in economic production which generally means smaller income. Assume then that the quantity w is chosen such that L is maximized. This appears to be a reasonable assumption because most work is dull, boring, troublesome and sometimes dangerous. One also can draw on a historical argument. In 1870 the yearly time spent in work was 2900 hours, in 1950 still 2000 but at present only 1600 on average. Simultaneously, life expectancy rose from 45 to almost 80 years due to the advances in medical sciences, nutrition and sanitary installations (which in this context must be considered as exogenous) and the GDP per capita increased from some 2000 PPPUS\$ well beyond 20000 PPPUS\$ due to higher productivity [46]¹. Higher life quality, therefore, was not only achieved through longer lives and higher consumption but also by significantly more leisure time. In fact, it has also been argued repeatedly on philosophical, sociological and economical grounds that time for leisure (under good economical conditions as well as in good health) is the ultimate source of life quality [95].

The real (i.e. net of inflation) gross domestic product (GDP) per capita and year is a common indicator of the economic status and behavior of a country. The GDP can be considered as a surrogate measure of wealth-related aspects of quality of life (as synonymous with standard of living). It is a suitable indicator of the economic aspects of quality of life. The GDP is roughly the sum of all incomes created by labor and capital (stored labor) in a country during a year. In most OECD countries about three quarters of the GDP are raised by labor and the rest by capital. Any transfer cost are carefully eliminated. Other equivalent definitions are: GDP = all expenditures on goods and services by the groups in society (households, businesses, all levels of government, foreigners), or GDP = production of goods by various industries - agriculture, manufacturing, wholesale, retail trade, etc.[7]. The GDP provides the infrastructure of a country, its social structure, its cultural and educational offers, the sustainability of the economy, its ecological conditions among others but also the means for the individual enjoyment of life by consumption in its various forms. In most developed countries about $60 \pm 10\%$ of the GDP is used privately, $15 \pm 5\%$ by the state (e.g. for military, police, jurisdiction, education, etc.) without transfer payments and the rest for (re-)investments. In most cases net exports can be neglected. The GDP also creates the possibilities to "purchase" additional life years through better medical care, improved safety in road and railway traffic, more safety in or around building facilities, more safety from hazardous technical activities, more safety from natural hazards, etc.. It does not matter whether those investments into "life saving" are carried out individually, voluntarily or enforced by regulation or by the state via taxes. Neither a minimum share for the state nor the investments into depreciating production means can

¹ All monetary values are given in international US\$ adjusted for purchasing power parity according to the World Bank [91].

be reduced appreciably. State consumption forms a large part of the (external) conditions for the individual to enjoy life in high quality, now and in the future. Continuing investments is a necessary condition for a sustainable economy. Therefore, only the part for private use is available for risk reduction as a minimum. The part available for risk reduction then is a little less than $g \approx 0.6$ *GDP* in first approximation and as a lower bound. A probably rather realistic estimate for g in terms of the *GDP* is obtained by leaving the investments untouched but take the government share without any transfer and subsidizing (social) payments giving $g \approx 0.7$ *GDP* or a little larger (see fig. 1 for an example [85]). An upper bound clearly is the *GDP* itself. The exact share for risk reduction must be determined separately for each country or group in a country and requires great care. The public must decide how much it is willing to spend on risk reduction, how much it is willing to give up of other public services and how much it wants to use for private consumption.

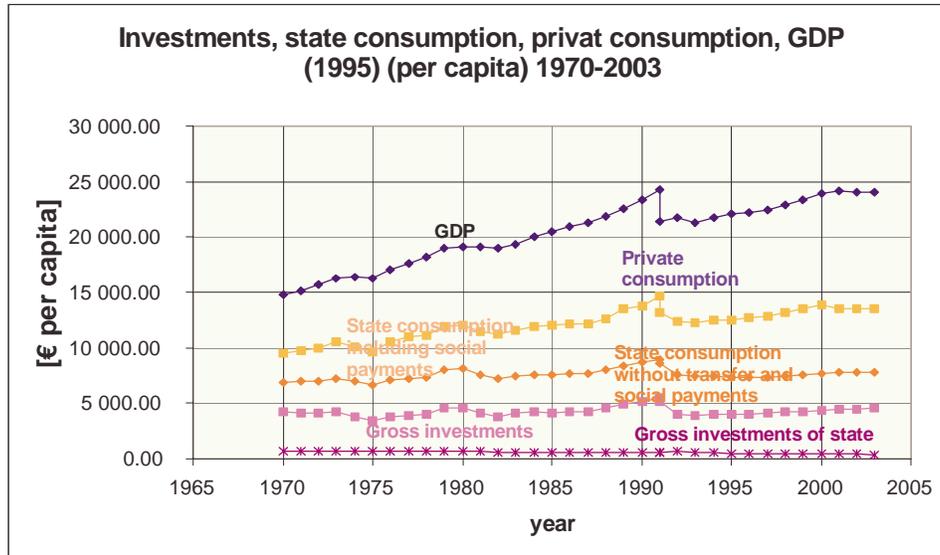


Figure 1: Gross investments, government consumption and private consumption for Germany

Some elegant mathematical derivation in Nathwani et al. [44] leads to the traditional form of the **Life Quality Index (LQI)**

$$L_w = g^w \ell^{1-w} (1 - w)^{1-w} \quad (1)$$

The fraction of time w of ℓ necessary for paid work varies between less than 0.1 and 0.25 (see [63] for estimates of w for different countries but also [51] and [46]). Eq. (1) is rederived in what follows. Nathwani et al. [44] start from a simple product $L = f(g)h(t)$ with $t = (1 - w)\ell$ the fraction of life devoted to leisure (or other non-productive time, e.g. sleep) and, consequently, $w\ell$ the fraction of life devoted to paid work. Thus, the LQI is a product of a function $f(g)$ measuring life quality and a function $h(t)$ measuring the duration of enjoyment of life. $f(g)$ and $h(t)$ are assumed to be independent, differential and monotonically increasing functions. Income and life expectancy are highly correlated across countries (see fig. 2). However, they are only weakly (and positively) correlated in countries with a developed social welfare system (see table 1 where it should be noted that the incomes in the highest quantile differ from those in the lowest quantile by a factor of 7 to 10 that is roughly the same range of incomes as the *GDP*'s in fig.2). The LQI as a product of two independent factors, therefore, is justified in good approximation.

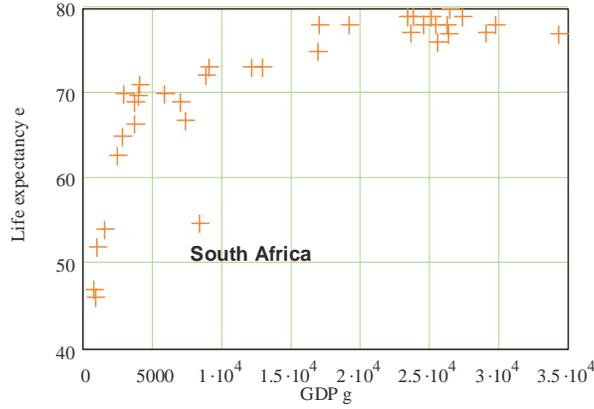


Figure 2: Life expectancy versus GDP in different countries [82]

	Germany			Canada	
Relative Income Position	Life expectancy Men	Life expectancy Female	Relative Income Position	Life expectancy	
1. Quartile	77	82	1. Quintile	74.0	
2. Quartile	82	85	2. Quintile	76.9	
3. Quartile	81	84	3. Quintile	77.5	
4. Quartile	83	86	4. Quintile	78.1	
			5. Quintile	78.5	

Table 1: Life expectancy versus income for Germany in year 2000 [69] and for Canada in year 1986 [54]

Introduce the differential $dL = \frac{\partial f(g)}{\partial g} h(t)dg + \frac{\partial h(t)}{\partial t} f(g)dt$ supposing that interest is in small changes in the LQI. Defining relative changes in the LQI by $\frac{dL}{L} = \frac{g}{f(g)} \frac{df(g)}{dg} \frac{dg}{g} + \frac{t}{h(t)} \frac{dh(t)}{dt} \frac{dt}{t} = k_g \frac{dg}{g} + k_t \frac{dt}{t}$ and setting $k_g/k_t = const.$ according to the universality requirement in [44] (indifference of the relative impact of g and t on life quality with respect to the actual values of g and t), one finds two differential equations $k_g \equiv \frac{g}{f(g)} \frac{df(g)}{dg} = r$ and $k_t \equiv \frac{t}{h(t)} \frac{dh(t)}{dt} = s$ with solutions $f(g) = g^r$ and $h(t) = t^s = ((1-w)\ell)^s$ ($r, s > 0$). This gives $L = g^r ((1-w)\ell)^s$ apart from proportionality constants. Nathwani et al. [44] then assumed $g \propto pw$ where p is proportional to labor productivity (monetary output per hour worked) and g is wealth production per year. Note the linearity of g on w . Other factors also affecting wealth production like capital, land and resources are merged into a proportionality constant.

”Presumably, people on the average work just enough so that the marginal value of wealth produced, or income earned, is equal to the marginal value of the time they lose when at work” as hypothesized by Nathwani et al.[44]. Standard economic models of labor supply also assume that the value of leisure time at the margin is equal to the marginal wage rate [7]. More directly, Lind [40] stated: *A prospect to save life or produce wealth is preferable in comparison with an alternative if the net increase in life expectancy is greater than its work time cost.* Consequently, people who produce wealth by work, possibly together with their families, optimize work and non-productive

time, i.e. their LQI. From $\frac{dL}{dw} = \frac{d}{dw}(pw)^r [(1-w)\ell]^s = \frac{r}{w} - \frac{s}{1-w} = 0$ for $\ell = \text{const.} > 0$ and $p = \text{const.} > 0$ one determines $r = s \frac{w}{1-w}$ which, without loss of generality, together with $r + s = 1$ results in $r = w$ and $s = 1 - w$ and

$$L_w = (pw)^r \ell^{1-r} (1-w)^{1-r} \approx g^r \ell^{1-r} \quad (2)$$

with $g = pw$. Also, it is $\frac{d^2L}{dw^2} \Big|_{r=1-s=w} < 0$ for $r, s > 0$. w ($w > 0$) can be assumed to be almost constant at a given time so that the factor $(1-w)^{1-r}$ can be dropped in many applications. Assuming that w is already at its optimum value w^* according to the work-leisure optimization principle just stated before we choose $r = w^*$. For later convenience we take the $(1-w^*)$ -th root in the numerator of eq. (1) and introduce the notation $q^* = \frac{w^*}{1-w^*}$. Additionally, we divide g^{q^*} by q^* . Dividing g^{q^*} by q^* removes a minor inconsistency of the original form for all practical purposes because persons with the same g and ℓ but larger w^* would have higher life quality (see also section 3). Apart from multiplicative constants we then have:

$$L_q = \frac{g^{q^*}}{q^*} \ell (1-w^*) \quad (3)$$

L_q fulfills the boundary conditions $L_q = 0$ for $g = 0$ and $\ell = 0$. L_q can and should be interpreted as a lifetime utility function $L_q = u(g, \ell, w)$ of an anonymous person. Viewed as a utility function the form eq. (3) has the desirable property of being risk-neutral with respect to life expectancy in contrast to eq. (2).

Nathwani et al.'s simple and straightforward reasoning has been scrutinized repeatedly by several investigators. One such investigation deserves special attention as it ends up with a slightly different form of the life quality index. Ditlevsen/Friis-Hansen [21] started from a general differential equation for the relative increments for the allocation of time for paid work $pw\ell$ and for leisure time $(1-w)\ell$, i.e. from $\frac{dA}{A} = f\left(\frac{d(pw\ell)}{pw\ell}, \frac{d((1-w)\ell)}{(1-w)\ell}\right)$ which must have the form $\frac{dA}{A} = c_1 \frac{d(pw\ell)}{pw\ell} + c_2 \frac{d((1-w)\ell)}{(1-w)\ell}$ (where after division by $c_1 + c_2$ it follows that new constants are $r = \frac{c_1}{c_1+c_2}$ and $s = \frac{c_2}{c_1+c_2} = 1-r$) with solution

$$L_{w\ell} = (pw)^r (1-w)^{1-r} \ell \quad (4)$$

as can easily be verified. r is some unknown combination coefficient. If this is determined by the work-leisure optimization principle, i.e. from $\frac{dL_{w\ell}}{dw} = \frac{d}{dw}((pw)^r (1-w)^{1-r} \ell) = 0$, one obtains $r = w^*$ with $g = pw^*$ as before. Similar to eq. (3) it is convenient to divide this equation by the exponent of g , w^* , so that finally

$$L_{w\ell} = \frac{1}{w^*} g^{w^*} (1-w^*)^{1-w^*} \ell \quad (5)$$

apart from proportionality constants. Note that for given p and ℓ eq. (4) or (2) are functions of w while eq. (5) or (3) are numbers in which $w = w^*$ is the actual (observed) value of the work fraction. Ditlevsen/Friis-Hansen denoted this new index by *Life quality time allocation index* (LQTAI). Risk-neutrality with respect to life expectancy comes out directly. Their derivation does not need to impose the constraint $k_g/k_t = \text{const.}$ but the constraint is implicitly present because r in eq. (4) must be constant. But they use the whole life for an allocation of work time and leisure time. If this is chosen in the derivations by Nathwani et al., i.e. if the work-leisure optimization principle is applied to the deterministic life time utility $L = (pw\ell)^r \ell^{1-r} (1-w)^{1-r}$, eq. (1) and (5) coincide. However, the derivation of the LQTAI does not directly allow an interpretation as a utility function. It is here where the concepts used by Nathwani et al. and Ditlevsen/Friis-Hansen differ. Otherwise, the two derivations are completely equivalent. Unfortunately, it will be almost impossible to verify

one or the other form empirically. The question which version to select will be taken up again in section 6².

The derivations can easily be extended to a non-linear relationship between g and w as pointed out recently by [56]. A non-linear relationship has been proposed in [20] but such relationships have a long history in economics [47]. The widely-used two-factor Cobb-Douglas production function represents any output, the output of a firm or the GDP in a macroeconomic context, as $Q = AK^\alpha L^\beta$ where A is a so-called technology constant, L is labor input, K is capital input and $0 < \alpha, \beta \leq 1$ in order to have diminishing returns in each variable accounting for the fact, that implementation of more labor or more capital becomes increasingly inefficient for given technology. At a given time A is exogenous and constant. A is not or only indirectly a controllable variable for decision-making. It depends on technological progress and how a society pushes it forward by clever use of labor and capital. Usually, $\alpha + \beta = 1$ is taken meaning constant returns to scale (doubling both inputs doubles output). $\alpha + \beta \geq 1$ implies increasing/decreasing returns to scale. The marginal product of capital is $MPK = \frac{\partial Q}{\partial K} = \alpha AK^{\alpha-1} L^\beta = \alpha \frac{Q}{K}$ and, similarly, the marginal product of labor is $MPL = \frac{\partial Q}{\partial L} = (1 - \alpha)AK^\alpha L^{\beta-1} = (1 - \alpha) \frac{Q}{L}$. Therefore, $\beta = 1 - \alpha$ can be computed as the ratio of labor output to total output because the labor share of total output is $MPL * L$. β turns out to be almost constant over the past decades with a slight trend to smaller β 's in recent years. This constancy is evidence for the validity of the Cobb-Douglas function. For example, for the US, Canada, Switzerland and Germany β is very close to 0.7 over the last 20 to 40 years (see fig. 3). In our notation, it is then $g \sim cw^\beta$ with $c = A(k)^\alpha$ ($k = \frac{K}{N}$, N = population size). Performing the same manipulations as before and collecting all constants in a new constant one finds for eq. (2)

$$r = \frac{w^*}{\beta - \beta w^* + w^*} \quad (6)$$

and, because $q = \frac{r}{1-r}$ with r in eq. (6),

$$q^* = \frac{1}{\beta} \frac{w^*}{1 - w^*} \quad (7)$$

in eq. (3). Although the validity of the Cobb-Douglas production function is discussed repeatedly in the literature there is good empirical evidence that it is at least a good approximation in many applications and for many countries.

The work-leisure optimization principle adopted before is central for the determination of the exponents in the two terms of the life quality index. Apart from the fact that the optimization principle makes sense intuitively, at least in the long run, and is clearly supported by the above-mentioned historical development of wealth, life expectancy and working time the crucial question is whether one can find other empirical evidence that the work-leisure optimization principle is indeed working. If it is active the second question is whether societies are already at the optimum. Some indirectly supporting material is given in [10] for European countries. For example, Bielski et al. [10] show by a representative inquiry of up to 3000 people in each of the different countries that people tend to prefer less work time in countries with a high GDP but would prefer more work and thus more income in countries with lower GDP (possibly with larger unemployment

² Ditlevsen/Friis-Hansen [21] define the optimal w as the value where $dp/dw = \infty$ with p a "productivity factor" being the GDP divided by the work based yearly salary. They deny the existence of the simple work-leisure optimization principle. Their criterion corresponds to the assumption that w in eq. (3) or (5) is already at a stationary value. Although the economical basis of the definition of p as a suitable parameter is doubtful it is remarkable that the results are very similar or even identical if the stationary or optimal value of w is determined by the same method (w = yearly working hours per wage earner/hours per year) in both cases. From a practical point of view it is, therefore, irrelevant which principle to use.

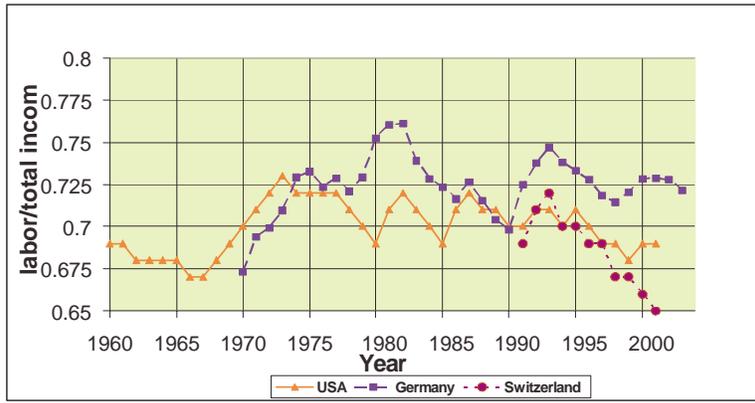


Figure 3: Cobb-Douglas Exponent for Labor Income for Germany, Switzerland and USA

rate and less work volume offered by the economy). Dependent full-time employees in countries with high GDP prefer to reduce their weekly working time from some 38.5 hours down to 34 to 36 hours. Self-employed people would like to reduce their work load from 48 hours down to 38 hours. The dependence between income and preferred weekly working time can be seen from the analysis in [10] on a household level (see table 2). In countries with a high GDP there is a clear tendency to redistribute the available work volume among a larger labor force (especially females with part-time work) in order to reduce the individual work load. In table 3 current and preferred working time is tabulated for some countries. If preferred working time is smaller than the actual one the income level is supposed to remain constant, if it is larger more income is desired. These tendencies are also supported by recent official labor statistics [51]. In [22] it is stated that in some countries full-time employees voted for weekly working times not below 35 hours.

	Actual situation	Preference
All couples	62 h	61 h
Financial situation		
● comfortable	66 h	61 h
● adequate	59 h	61 h
● difficult	53 h	64 h

Table 2: Actual and preferred weekly working hours for households [10]

The decline in working hours for full-time work which can be traced back for over a century is slowed down or even stopped in recent years [51] (see fig. 4). In the long run the decline is a little less than linear with time. This tendency is also predicted from microeconomic considerations, for example, in [7]. But further slow reductions in yearly working hours are still to be expected in many countries once the transition from purely full-time work to part-time work is realized. On the other hand, economists predict a countercurrent trend in the future due to the increase in life expectancies in the next few decades. There are two exceptions from these trends which are the USA and Sweden, for two completely different reasons, however. For the USA a growing part of over-time work in a fast growing economy (growth rate 2.8%) has led to a slight increase in working time since 1980 (without increasing part-time work). The relatively large annual working times are also partially due to the household survey technique as opposed to the establishment survey technique used in most other countries [50]. In Sweden it was just the large increase in part-

time work together with rather flexible (legalized) rules for individual work-time preferences [51]. Therefore, Sweden is in line with the work-leisure optimization hypothesis while the development in the USA is exceptional.

It seems as if the actual work load (and the corresponding income) is about the preferred work load possibly except for Portugal, Spain and the United Kingdom (see table 3).

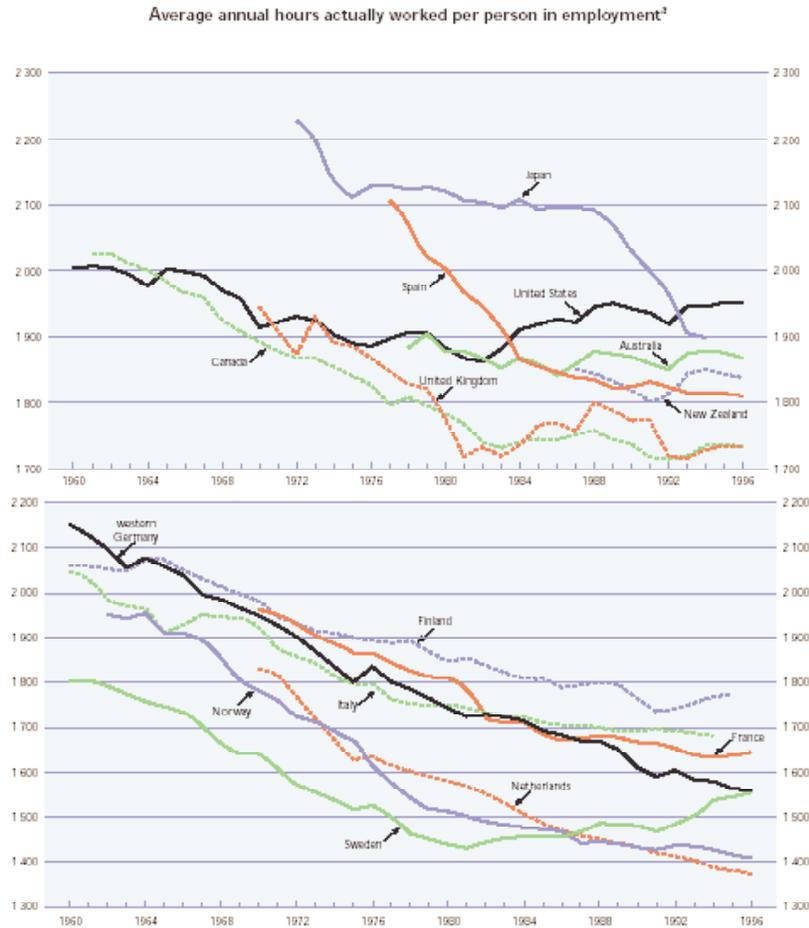


Figure 4: Average-yearly-worked-hours

It is clear that many factors determine the actual and preferred work load for individuals and for households in the various countries including traditions, cultural aspects, the social environment, strength and role of trade unions, shares of dependent employment and self-employment, female participation in the labor force, legal conditions, general economic performance and, not the least, personal and societal preferences. Whether the unemployment rate plays an important role for the actual and preferred working time appears to depend to a large extent on the subsistence level for the unemployed. The same appears to be true whenever the preferred working hours are much larger than the actually worked hours. But the general trend of working less when the GDP is at a high level and is growing is very obvious. On the other hand, low incomes relative to the incomes in richer countries and within a country lets people prefer to work more and get more income given the productivity in a country. The work-leisure optimization principle must be considered as effective in general and in the long run. Empirical evidence that countries are already at the

optimum, however, is difficult to find. It would imply that the work time fraction has stabilized. But it is unknown whether it ever fully stabilizes.

Country	GDP in PPP US\$	Growth rate per capita in %	Unemploy. rate in %	Part-time employ. in %	Aver. current weekly hours	Aver. preferred weekly hours	Differ. in %
Austria	26310	2.0	5.4	12.6	66.6	62.1	-6.8
Belgium	27500	2.2	8.4	14.0	65.4	62.0	-5.2
Denmark	25500	1.6	5.3	21.5	68.5	61.8	-5.5
Finland	22900	2.0	9.8	9.9	67.7	66.3	-2.1
France	24470	1.7	9.7	14.7	62.4	66.2	+6.1
Germany	25010	1.9	9.9	17.1	60.8	59.6	-2.0
Greece	16900	0.9	11.3	4.3	65.1	67.3	+3.4
Ireland	25470	4.0	4.1	18.3	61.8	58.3	-5.7
Italy	23400	2.1	10.4	11.8	58.0	58.9	+1.5
Luxembourg	36400	3.9	2.7	7.6	58.0	55.8	-3.8
Netherlands	26170	1.8	2.6	30.4	58.3	55.9	-4.1
Portugal	17000	2.9	4.3	9.3	59.1	70.8	+19.7
Spain	19300	2.2	14.0	7.9	54.4	66.0	+21.1
Sweden	23770	1.4	6.0	14.5	69.3	65.9	-5.0
United Kingdom	23500	2.0	5.5	23.0	66.4	58.9	-11.3
Norway	29760	2.6	3.0	20.7	66.4	66.2	-0.4

Table 3: Actual and preferred weekly working hours of both partners in a couple (household) after [10].

The life quality index $L = (cw^\beta)^r((1-w)\ell)^{1-r}$ shows a rather flat optimum if plotted against w for given (optimal) exponent $r = 0.13$, ℓ and g (see fig. 5). In other words, there is room for changing w without changing the LQI significantly. This may also serve, at least in part, as an explanation for minor variations in w between countries under otherwise similar conditions. Nevertheless, there are good reasons to believe that most countries given their specific conditions are already at or close to the optimum.

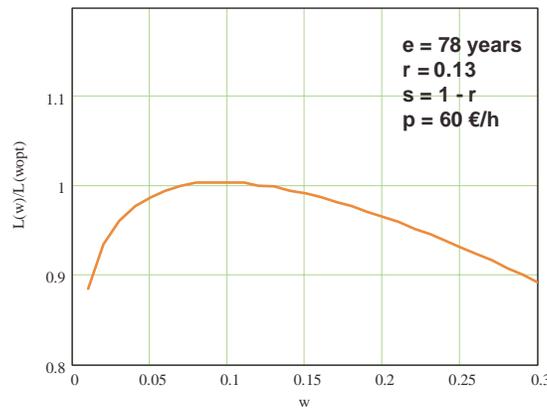


Figure 5: LQI as a function of w for fixed exponents

Finally, it should be emphasized that all quantities entering eq. (3) must be mean values due to

the constitutional equality principle. In the following we prefer eq. (7) in eq. (3) but later result are easily re-computed for other forms of the LQI. In the following all further derivations are carried out using the general parameter q^* . The asterix denoting optimal values will be dropped.

In summary, it is remarkable that all fundamental assumptions in the original developments by Nathwani et al. [44] are well supported by data, at least to the extent one can expect in this complex and difficult field. Some more discussion can be found in [65] and [66].

3. LQI as a Substitute for the HDI

The LQI initially was developed to be used as an alternative for the human development index (HDI) [?] as a measure of "how well a nation serves the well-being of its citizens". Then, it must be calculated with the full GDP, of course. Fig. 6 displays various versions of the index over the (optimum) w . It is seen that the original LQI in eq. (1) divided by w fulfills the reasonable requirement that more work decreases life quality, at least within the range of w to be expected in practice. It is also in this context that Nathwani et al.'s ethical requirement of indifference of the relative impact of g and t on life quality with respect to the actual values of g and t makes good sense. But the versions eq. (3) or eq. (3) with (7) are not suitable while the original version with Cobb-Douglas correction before taking the $\frac{w}{\beta-\beta w+w}$ -th root is again acceptable, i.e.

$$L_{w,\beta} = \frac{g^{\frac{w}{\beta-\beta w+w}}}{\frac{w}{\beta-\beta w+w}} \ell^{1-\frac{w}{\beta-\beta w+w}} (1-w)^{1-\frac{w}{\beta-\beta w+w}} \quad (8)$$

Also, if used in this context the LQI should be appropriately normalized (see also [41]). Unfortunately, there is no obvious procedure to normalize it.

4. Acceptability criteria

Shepard/Zeckhauser and others [75] [39] derived the *willingness-to-pay* from the condition that a change in life expectancy and the corresponding change in consumption balance each other by keeping the life quality index (or the remaining life time utility) constant, $L = L(\ell, g) = \text{const.}$ or, equivalently,

$$dL = \frac{\partial L}{\partial \ell} d\ell + \frac{\partial L}{\partial g} dg = 0 \quad (9)$$

so that after rearrangement

$$WTP = dg = -\frac{\frac{\partial L}{\partial \ell}}{\frac{\partial L}{\partial g}} d\ell$$

Inserting eq. (3) leads to

$$dg = -\frac{g}{q} \frac{d\ell}{\ell} \quad \text{or} \quad \frac{dg}{g} + \frac{1}{q} \frac{d\ell}{\ell} \geq 0 \quad (10)$$

Criteria of the type (10) remain unaffected by multiplicative constants such as the productivity p or multiplicative corrections of life expectancy as proposed in health related economic studies in order to adjust life expectancy for lifetimes in bad health (which very well enter numerically into the LQI). Also, those criteria are independent of monotone transformations as done, for example, in eq. (3). Eq. (9) sometimes is also called the "LQI invariance principle". The equality in (10) gives an indication of what is **necessary** and **affordable** to a society for life saving undertakings; projects having " $<$ " are not admissible. The latter projects would, in fact, be life-consuming and,

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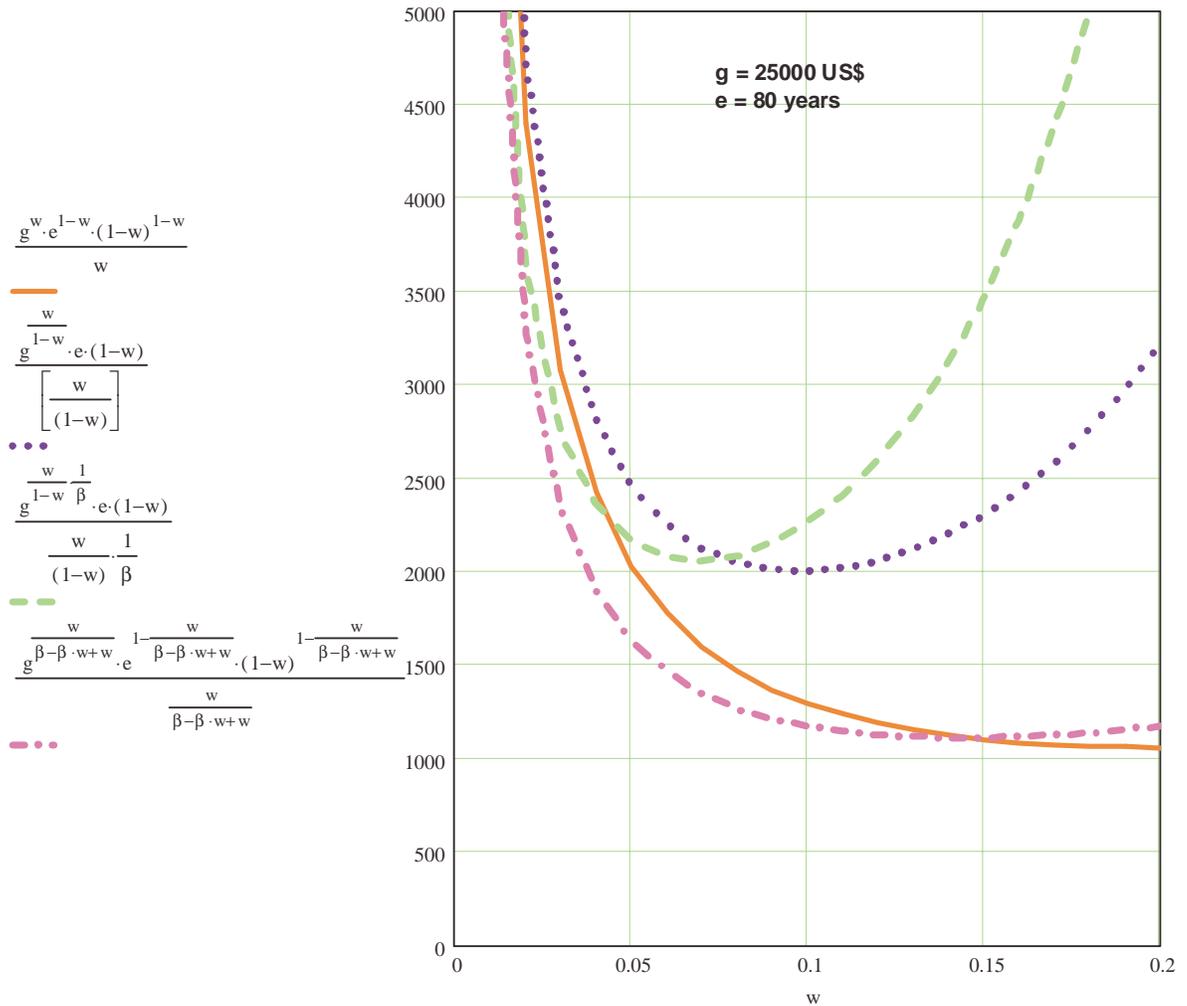


Figure 6: Different versions of the LQI ($g = 25000$ US\$, $\ell = 80$ years)

thus, be in conflict with the constitutional right to life. Whenever a given incremental increase in life expectancy by some life saving operation (positive $d\ell$) is associated with larger than optimal incremental cost (negative dg) one should invest into alternatives of life saving. If a given positive $d\ell$ can be achieved with less than required by eq. (10) it should be done, of course. Eq. (10) is easy to interpret. For example, for a 1% increase in life expectancy yearly investments of about 5% of g for $q = 0.2$ would be affordable. From a practical point of view it is important that all quantities on the left-hand side of eq. (10) are easily available and can be updated any time. The democratic equality principle dictates that average values for g , ℓ and w have to be taken. Any deviations from average values for any specific group of people need to be justified carefully if eq. (10) is applied to projects with involuntary risks. It is important to note that the simple criterion (10) is independent of any benefit derived from the life saving undertaking other than life saving and in so far is also independent of any discounting.

5. Sustainable Discounting

In view of the time horizon of some 20 to more than 100 years (i.e. several generations) for civil engineering facilities discount rates should be long-term averages. They should be net of inflation and taxes. Investments into risk reduction must be discounted as other investments in order to avoid inconsistencies [58]. In the private sector a long term real interest rate is roughly identical to the (maximum) return rate one could get from an investment. But can the public also adopt such a strategy? The public does not make financial profit except by its economical growth. Public interest rates vary greatly among countries and appear to be set according to quite different rationales. For information, table 5 gives some discount rates used by government agencies in different countries. These rates are meant for comparing alternatives. This table clearly shows that there is hardly a common rationale behind setting public interest rates.

Country	Real discount rate in %	Projects	Sources
Australia	7 %		[Val/Stewart, 2003]
Austria	<i>Actual market rate</i>	Road construction	
Denmark	6 - 7	Road construction	1) Road directory: http://www.vejregler.dk/pls/vrdad/vr_layout.vis?p_gren_id=5369 2) Transportat: http://www.femer.dk/indhold/docs1/k13_006.htm
Finland	6		[Val/Stewart, 2003]
France	8	Public works	Commissariat général du Plan, 2000
Germany	3	Public works	1) Bundesministerium für Verkehr, Bau- und Wohnungswesen, Bundesverkehrswegeplan 2003 2) LAWA (Länderarbeitsgemeinschaft Wasser): Leitlinien zur Durchführung von Kostenvergleichsrechnungen, 1993
Netherlands	4	Dikes	Kabinetsstandpunt Heroverweging Disconteringsvoet 1995
Sweden	4		[Val/Stewart, 2003]
Switzerland	2 - 4	Public works	
UK	6 ¹⁾ , (4 ¹⁾ , 8 ²⁾	general	1) HSE (Health & Safety Executive): Reducing risks, protecting people. HSE's decision-making process, 2001 → guidance document HSE, 2001: 4% 2) UK Dept. of Transportation, [Val/Stewart, 2003]
USA	7 ¹⁾ , 2 - 3 ²⁾	Highway safety	1) U.S. Dept. of Transportation, Federal Highway Administration, 1994, http://www.fhwa.dot.gov/legisregs/directives/techadvts/t75702.htm 2) [Val/Stewart, 2003]

Table 4: Public interest rates

Recently, much discussion has been taken place in the context of sustainable development, long term public investments in general and intergenerational justice - aspects which appear very relevant in our context. Weinstein/Stason [92] and others require that interest rates for life saving interventions should be the same as for other cost and thus equal to the real market interest rate, simply for consistency reasons. This appears to be an extreme point of view. The other extreme of not discounting intergenerationally at all is expressed, for example, in [14] and [72], based primarily on ethical grounds in the context of CO₂-induced global warming, nuclear waste disposals, depletion of natural resources, etc.. But greenhouse gas abatement and other long term public investments are probably somewhat different from our goal of risk reduction as pointed out in [73]. But discounting must be considered as a fact of life and modern economic mechanisms [60].

The real public interest rate is strongly related to the benefit a society earns from the various

activities of its members, i.e. its real economic growth per capita (see also [63] where the interest rate has been set equal to the growth rate). It is important to take it per capita because it then reflects the counteracting effects of economic and population growth. In the economics literature this is sometimes called the "natural interest rate". The United Nations Human Development Report 2001 [?] gives values between 1.2 and 1.9 % for industrialized countries during 1975-1998. If one considers the last 120 years and the data in [46] for some selected countries one determines a growth rate per capita of about 1.8% for Western Europe and the so-called Western Offshoots, USA, Canada and Australia, and a little more for Japan. The long time period of 120 years is justified as it covers the period of industrialization including some economic crisis and several times of war and postwar years. The same period is also approximately the time for which risk reducing interventions should be valid.

Modern economic growth theory has been widely applied to sustainability financing. Nordhaus [45] and others (see [81] for an overview but also the other papers in Energy Policy, 23, 3/4, 1995 as well as Int. J. Sustainable Development, 6,1, 2003) follow the classical Ramseyan approach (see [68] [77] and [8]) for optimal stable economic growth in perfect markets

$$\gamma = \rho + \epsilon\delta > 0 \quad (11)$$

where γ is the real market interest rate, $\rho > 0$ the rate of pure time preference of consumption, ϵ the elasticity of marginal consumption (income) and δ the consumption (income) growth rate per capita. Clearly, the subjective element is the quantity ρ . The rate ρ has been interpreted as the effect of human impatience, myopia, egoism, lack of telescopic faculty, etc.. Its existence in human behavior has been widely demonstrated in human ethology and economics [60] . It is partially justified because there is uncertainty about one's future. However, there are many authors in economics as well as philosophical and political sciences including Ramsey who refuse convincingly to accept a rate $\rho > 0$ in intergenerational contexts on ethical grounds (see, for example, [14] [72] [61]) while it is considered fully acceptable for intragenerational discounting. On the other hand, intergenerational equity arguments in Arrow [2] indicate that there should be a $\rho > 0$ in order to remove an "... *incredible and unacceptable strain on the present generation*" Rabl [61] argues that there must be $0 < \gamma < \epsilon\delta$ in the framework of long-term public investments. In [64] simple, intergenerationally acceptable bounds are given for ρ and γ . In the light of the following considerations these bound are not fully convincing.

In the economic and ecological literature the adequacy of the Ramseyan model has been seriously questioned. So-called overlapping generation models or generation adjusted discounting models are advocated instead. The main idea is to discount for living generations at any time at the rate in eq. (11) with $\rho > 0$, i.e. with $\gamma = \rho + \epsilon\delta$, but diminish the rate for all yet unborn generations down to $\epsilon\delta$, thus also facilitating the transition into a sustainable state of economy. Accordingly, Bayer [9] assumed that each generation follows its own preferences, thus discounts with γ including the myopic part and distributes consumption (loss) effects evenly among its members. For yet unborn generations discounting is only done by $\epsilon\delta$ and the requirement of intergenerational equity is fulfilled. This model, slightly modified in [67] , assuming the same preferences for all generations then produces an equivalent time-variant discount rate (discrete discounting) for a consumption effect c_m at time m

$$\gamma'_E(m) = \left(\frac{c_m}{PV_{OLAG}(m)} \right)^{1/m} - 1 \quad (12)$$

where the present value for this consumption effect for overlapping age groups (OLAG) can be

determined from

$$PV_{OLAG}(m) = \begin{cases} \sum_{a=0}^{m-1} \frac{c_m h(a,n)}{(1+\epsilon\delta)^{m-a}(1+\rho+\epsilon\delta)^a} + \sum_{a=m}^{a_u} \frac{c_m h(a,n)}{(1+\rho+\epsilon\delta)^m} & \text{for } m \leq a_u \\ \sum_{a=0}^{a_u} \frac{c_m h(a,n)}{(1+\epsilon\delta)^{m-a}(1+\rho+\epsilon\delta)^a} & \text{for } m > a_u \end{cases} \quad (13)$$

a_u is the largest age considered in the life table and $h(a, n)$ the age distribution of the population ($n =$ population growth rate). $\gamma'_E(m)$ can be shown to converge to $\epsilon\delta$ for $m \rightarrow \infty$. The rate eq. (12) can well be approximated by

$$\gamma'_E(m) \approx \epsilon\delta + \rho \exp[-am] \quad (14)$$

with $a (\approx 0.013)$ a suitably chosen constant. The discount rate for continuous discounting is $\gamma_E(t) = \ln(1 + \gamma'_E(t)) \approx \gamma'_E(m)$. Fig. 7 shows that the equivalent discount rate approaches its asymptotic value after practically 200 to 300 years (solid line). The dotted line represents the discount rate without weighting by the age distribution. Clearly, the critical component still is the equivalent time preference rate function $\rho \exp[-am]$, or more specifically, the magnitude of ρ , which must be selected in accordance with economical market mechanisms. International organisations frequently determine realistic constant rates γ from long-term rates referring to secondary market yields on long-term bonds which range between 3 to 6% for most countries. Similar and in part stronger arguments than in [9] for a decaying interest rate have been put forward in [25].

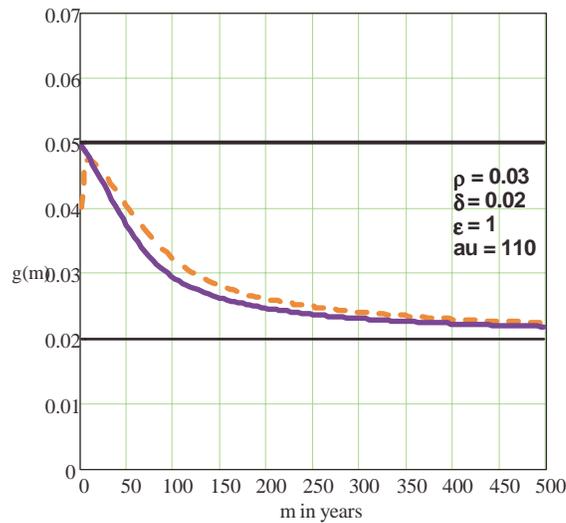


Figure 7: Equivalent time-dependent discount rate according to the OLAG-model

For any discounting scheme the discount rate should be the smallest possible. Weitzman [93] and others, in fact, showed that the far-distant future should be discounted at the lowest possible rate > 0 if there are different possible economic scenarios each with a given probability of being true (see also [25]). This strategy appears to be highly recommendable in view of the present overuse of non-renewable natural resources (see also [67]).

6. Discounting Utilities and Age-averaging

We are now going to imbed the foregoing derivation for the life quality index into a more general setting. Health-related economics have developed similar concepts independently and in part ear-

lier starting with the seminal work by Usher [83] . The expected remaining present value life time utility at age a (conditional on having survived until a) for $\gamma(t) = \rho(t) + \delta$ is (see, for example, [94] [1] [74] [75] [70] [33] [34])

$$\begin{aligned}
L(a) &= E[U(a)] = \int_a^{a_u} \frac{f(t)}{S(a)} U(a, t) dt \\
&= \int_a^{a_u} \frac{f(t)}{S(a)} \int_a^t u[c(\tau)] \exp \left[- \left(\int_a^\tau \gamma(\theta) d\theta \right) \right] d\tau dt \\
&= \frac{1}{S(a)} \int_a^{a_u} u[c(t)] \exp \left[- \left(\int_a^t \rho(\theta) d\theta + \delta(t - a) \right) \right] S(t) dt
\end{aligned} \tag{15}$$

where $f(t)dt = \left(\mu(t) \exp \left[- \int_0^t \mu(\tau) d\tau \right] \right) dt$ is the probability of dying between age t and $t + dt$ which can be computed from life tables containing the age-dependent mortalities $\mu(t)$ and $S(t)$ is survival probability ³. a_u is some maximum possible lifetime which is taken as the maximum age considered in modern life tables. Introduction of a constant consumption rate c independent of t , which can be shown to be optimal under perfect market conditions [75] , simplifies eq. (15) greatly ⁴.

$$L(a) = u[c] \ell_d(a, \rho, \delta) \tag{16}$$

where $\ell_d(a, \rho, \delta)$ is the "discounted" life expectancy at age a

$$\begin{aligned}
\ell_d(a, \rho, \delta) &= \frac{1}{S(a)} \int_a^{a_u} \exp \left[- \left(\int_0^t \mu(\tau) d\tau + \int_a^t \rho(\tau) d\tau + \delta(t - a) \right) \right] dt \\
&= \int_a^{a_u} \exp \left[- \left(\int_a^t (\mu(\tau) + \rho(\tau)) d\tau + \delta(t - a) \right) \right] dt
\end{aligned} \tag{17}$$

The expected remaining present value life time utility $L(a)$ decays with age as $\ell_d(a, \rho, \delta)$. We now

³ In economics the function (15) is used to find the optimal consumption path $c^*(t)$ by dynamic optimization [31] given a realistic function for the earnings, rational behavior of the consumer and subject to the budget constraint

$$\dot{k}(t) = rk(t) + i(t) - c(t) - f(t); k(0) = k(a_u) = 0$$

where $t \geq a$, $k(t)$ denotes assets at age a , r = the real (market) interest rate, $i(t)$ the earnings per time unit, $c(t) \geq 0$ the consumption and $f(t)$ the receipts or payments of a (fair) insurance (see, for example, [75] [34]).

⁴ Shepard/Zeckhauser [75] assume that there are no legacies or bequests. At birth there is no wealth and there is no wealth left at death at some large age. During years of no or low earnings the individual can borrow (life-insured) money against future earnings and can invest into (fair) life insurances during periods of larger earnings in order to cover the expenses after retirement. They find that the maximum and optimal consumption level is constant over the lifetime

$$c = \frac{\int_0^{a_u} \exp \left[- \int_0^\tau \gamma(\theta) d\theta \right] m(\tau) S(\tau) d\tau}{\int_0^{a_u} \exp \left[- \int_0^\tau \gamma(\theta) d\theta \right] S(\tau) d\tau}$$

where $m(\tau)$ are the earnings, i.e. it is the ratio of discounted lifetime earnings to discounted life expectancy.

In the real world perfect markets do not exist. Shepard/Zeckhauser therefore considered also the other extreme, the so-called "Robinson Crusoe" case, where the individual is absolutely self-consistent leading to a somewhat different consumption pattern and age-dependency of $L(a)$. They claim that reality is somewhere in between. Since we are considering the whole life an individual we may say slightly more realistically that the family and possibly the public provides support for consumption during childhood and education and life insurances, public or other pension systems and/or income support programs provide sufficient support during retirement (see also [12]). This acts implicitly as the supposed perfect market. The assumption of constant consumption throughout one's lifetime, therefore, will be valid at least in good approximation.

use a utility function well-known in economics

$$u[c] = \frac{c^q - 1}{q} \approx \frac{g^q}{q} \quad (18)$$

with $c \approx g \gg 1$ (see also eq. (3) in which the monetary part has exactly the same form as eq. (18)). Eq. (18) (or if eq. (3) is interpreted as an utility function) it belongs to the class of so-called iso-elastic constant relative risk aversion (CRRA) functions according to Arrow-Pratt. In [54] an extensive discussion about the meaning of the value of q in an economic context is provided. The risk aversion parameter is $\epsilon = 1 - q$ for $\frac{g^q}{q}$ where $\epsilon = -\frac{g \frac{d^2 u(g)}{dg^2}}{\frac{du(g)}{dg}}$ is the elasticity of marginal consumption. The parameter q must reasonably be bounded ($0 \leq q \leq 1$) because there should be $\frac{du(g)}{dg} = g^{q-1} > 0$ for $q \geq 0$ and all consumption levels $g > 0$ and $\frac{d^2 u(g)}{dg^2} = (q-1)g^{q-2} < 0$ for $q < 1$ implying concavity of $u(g)$. A large q indicates low risk aversion and preference for large consumption g and vice versa.

It appears natural to use values for q consistent with the work-leisure optimization principle as outlined and verified above. The utility considerations suggest that the only reasonable version of the LQI is to use eq. (3) with eq. (7)⁵. The parameter q as determined by eq. (7) turns out to be around $0.14 \pm 50\%$. It should be mentioned that economists determined the "risk aversion parameter" q in the order of 0.2 (see, for example, [1], [75], [70]) and could support it in tendency by some empirical studies (mostly by contingent valuation studies). Studies on intertemporal consumption choices are not directly relevant for our purposes. Also, in a recent study treating q as a fitting parameter in time series for life expectancy and GDP values between 0.07 and 0.21 have been determined for some countries[30].

By averaging over the age-distribution of a (demographically stable or stationary) group

$$h(a, n) = \frac{\exp[-na] S(a)}{\int_0^{a_u} \exp[-na] S(a) da} \approx \frac{S(a)}{\ell(0)} \quad (19)$$

Pandey/Nathwani [54] [55] defined the *societal life quality index* corresponding to eq. (3) as

$$SLQI(g, q, \bar{\ell}) = \frac{g^q}{q} \int_0^{a_u} \ell_d(a, \rho, \delta) h(a, n) da = \frac{g^q}{q} \bar{\ell}(\rho, \delta) \quad (20)$$

with

$$\bar{\ell}(\rho, \delta) = \int_0^{a_u} \ell_d(a, \rho, \delta) h(a, n) da$$

and which also is to be interpreted as an age-averaged expected remaining present value life time utility. The assumption of a stable population is only an approximation. Age-averaging is necessary if one considers event-type hazards affecting a representative cross-section of the population.

Using the equivalent of the indifference relation eq. (9) then yields an acceptability criterion

⁵ In order to see this we neglect discounting and age-averaging. Using eq. (15) for $a = 0$ without discounting and with $u[c(t)] = u[c]$ yields

$$L(0) = u[c] \int_0^{a_u} S(t) dt = u[c] \ell(0)$$

as the undiscounted, expected lifetime utility. It follows that c must be a yearly rate so that $c = g = pw$ (or $c = g = cw^\beta$) is the correct form to use for eq. (2). The LQI in the form eq. (3) as a utility function is thus recovered from utility considerations. In a utility context the form eq (5) is not admissible.

corresponding to eq. (10) and which is denoted as "societal willingness-to-pay"

$$\frac{dg}{g} + \frac{1}{q} E_A \left[\frac{d\ell_d(A, \rho, \delta)}{\ell_d(A, \rho, \delta)} \right] \geq 0 \quad \text{or} \quad dg \geq -\frac{g}{q} E_A \left[\frac{d\ell_d(A, \rho, \delta)}{\ell_d(A, \rho, \delta)} \right] \quad (21)$$

It is possible and many times useful to replace the quantities $\frac{d\ell}{\ell}$, $\frac{d\bar{\ell}(\rho, \delta)}{\bar{\ell}(\rho, \delta)}$ or $\frac{d\ell_d(a, \rho, \delta)}{\ell_d(a, \rho, \delta)}$ in criteria of the type eq. (10) or (21) by simple difference quotients with y a small number or by Taylor expansions to first order (at $x = 0$) (see for example [29])

$$E_A \left[\frac{d\ell_d(A, \rho, \delta)}{\ell_d(A, \rho, \delta)} \right] \approx E_A \left[\frac{\ell_d(A, \rho, \delta, y) - \ell_d(A, \rho, \delta, 0)}{y} x \right] \approx E_A \left[\frac{\frac{d}{dx} \ell_d(A, \rho, \delta, x) |_{x=0} x}{\ell_d(A, \rho, \delta)} \right] \approx -C_x(\rho, \delta) x \quad (22)$$

for small mortality reductions. The quantity

$$G_x = \frac{g}{q} C_x(\rho, \delta) \quad (23)$$

is the monetary amount a society should be willing to pay for a unit mortality reduction. It is further discussed below.

For example, for a mortality reduction scheme reducing mortality by a constant small quantity Δ , i.e. $\mu_\Delta(a) = \mu(a) + \Delta$ one finds

$$C_{\Delta\bar{\ell}}(\rho, \delta) = - \int_0^{a_u} \frac{\int_a^{a_u} (t-a) \exp \left[- \left(\int_a^t (\mu(\tau) + \rho(\tau)) d\tau + \delta(t-a) \right) \right] dt}{\int_a^{a_u} \exp \left[- \left(\int_a^t (\mu(\tau) + \rho(\tau)) d\tau + \delta(t-a) \right) \right] dt} h(a, n) da \quad (24)$$

The index "x" in eq. (22) stands for the leading parameter in a specific mortality reduction scheme, for example, whether the reduction is constant independent of age (regime Δ), proportional to the age-dependent mortalities (regime π) or affecting only special ages (regime α). C_x relates (small) changes in mortality of that specific scheme to (small) changes in life expectancy. Examples are $\mu_\pi(\tau) = \mu(\tau)(1 + \pi)$, $\mu_\Delta(\tau) = \mu(\tau) + \Delta$ or $\mu_\alpha(\tau) = \mu(\tau) + \begin{cases} \Delta & \text{for } \tau > 60 \\ 0 & \text{else} \end{cases}$. The constant

C_x depends on the specific mortality reduction scheme, i.e. the age dependency of a mortality reduction (see [65] , [64] , [33]) but also on the operations performed (discounting and/or age-averaging). The influence of such schemes can be significant. Relationships like eq. (22), of course, also hold if no discounting and no age-averaging is performed. For the examples just given, there is for a European life table $C_\pi \approx 15$, $C_\Delta \approx 45$ and $C_\alpha \approx 24$, respectively, if no discounting and age-averaging is performed. Performing age-averaging alone gives $C_\pi \approx 28$, $C_\Delta \approx 24$ and $C_\alpha \approx 3$ while discounting alone gives $C_\pi \approx 4$, $C_\Delta \approx 24$ and $C_\alpha \approx 6$. However, if discounting and age-averaging is applied we have $C_\pi \approx 18$, $C_\Delta \approx 17$ and $C_\alpha \approx 2$. The two types of operation obviously work in the same direction. Only a scheme with a constant mortality reduction at all ages as assumed in eq. (24) is intragenerationally acceptable⁶. For a given mortality reduction scheme eq. (22) is invertible, i.e. for a given mortality change x we also can determine the associated

⁶ Intragenerational equity is shown as follows. Assume that a representative portion of the population is exposed to a hazard, i.e. all are trapped in the event. In the event the crude mortality changes by dm . Using $q(a) \approx m(a)$ where $m(a)$ is the age-dependent death rate (number of deaths in the age interval $[a, a + da]$ divided by number of people at age a it is clear that a given number N_F of fatalities (or averted fatalities) distributes itself among ages according to the age distribution of the population, i.e. $n_F(a) = N_F h(a, n)$. But the number of people at age a is also $n(a) = N h(a, n)$ with N the population size. Therefore, $dm(a) = \frac{n_F(a)}{n(a)} = \frac{N_F}{N} = \text{constant}$. No other mortality reduction scheme shares this property.

relative change in life expectancy. If mortality is a function of some other quantity x can also represent the leading parameter in that quantity. The quantity $\frac{d\ell}{\ell}$ is in rough numerical agreement with an approximation proposed in [20] .

In health-related economics the *value of a statistical life (VSL)*⁷ has been defined using the *willingness-to-pay (WTP)*, implicitly assuming that the following relationship holds:

$$WTP(a) = \frac{g}{q} \frac{d\ell_d(a, \rho, \delta)}{\ell_d(a, \rho, \delta)} \quad (25)$$

An indifference relation similar to eq. (9) is also adopted. This has been discussed thoroughly in [34] and [12] , especially the conditions under which eq. (25) is valid. For constant consumption over the lifetime and a constant mortality reduction at all ages, i.e. $dm = \Delta$, eq. (25) is, in fact, a correct definition. It can be derived from the life quality index by dividing it by the marginal utility $u'(g) = g^{q-1}$ (in order to obtain a monetary quantity). Performing age-averaging one can define the *societal value of a statistical life (SVSL)*, i.e.

$$SVSL = \frac{g}{q} \bar{\ell}(\rho, \delta) \quad (26)$$

and, correspondingly, the *societal willingness-to-pay (SWTP)*

$$SWTP = E_A \left[\frac{g}{q} \frac{d\ell_d(A, \rho, \delta)}{\ell_d(A, \rho, \delta)} \right] \quad (27)$$

which, under otherwise the same conditions, is identical with eq. (21).

Both lines of thought, the economical (*SVSL*) and the LQI approach (*SLQI*), have a good conceptual and theoretical basis. They complement each other. In particular, the derivations for eq. (2) justify the power function form in eq. (18) and lets eq. (20) be interpreted as an expected remaining present value life time utility for all those alive at $t = 0$. Neither criterion (21) nor (27) depend on any benefit other than risk reduction or life extension. In most applications clear support for decisions can be reached by using either of the approaches, even the one without discounting and age averaging. Age averaging is generally necessary for the technical applications we have in mind because the risk reduction intervention is to be executed at $t = 0$ for all those living now. The concept of discounting future utilities by $(\rho + \delta)$ may be debatable as the subjective time preference rate ρ is concerned but not with respect to the population and economic growth. The *SLQI*–based approach explicitly combines three important human concerns, that is high life expectancy, high consumption and an optimized time available for the development of one’s personality off the time for paid work. Criterion (21), having in mind its derivation, also tells us that larger expenses for risk reduction are inefficient and smaller expenses are not admissible in view of the constitutional right for life. In particular, criterion (21) is affordable from a societal point of view.

7. Application to Technical Facilities under Event-type Hazards

7.1 Cost-benefit Analyses

In applying criteria of the type (21) to technical facilities under event-type hazards (natural or man-made) two aspects have to be considered, the cost benefit aspect and the public safety aspect.

⁷ To assign a monetary value to human life, on whatever basis, to a known or anonymous, young or old, rich or poor, healthy or sick individual is a very controversial issue. In fact, a monetary value of life does not exist. "...the value of human life is infinite and beyond measure, ..." [35] . It must rather be understood as a formal constant in a relation expressing the "willingness-to-pay" for risk reduction given the financial capabilities of a society.

Below three representative objective functions will be given.

For systematic reconstruction after failure, negligibly short reconstruction times and single mode failure an appropriate objective function based on the renewal model is

$$Z(\mathbf{p}) = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F)h^*(\gamma, \mathbf{p}) \quad (28)$$

which is to be maximized. In this function b is the (here constant) benefit per time unit, γ the (here constant) long term market interest rate for continuous discounting, $C(\mathbf{p})$ the construction cost resp. the reconstruction cost, H_M the physical damage cost, $H_F = kN_{PE}(SHC \text{ or } SLSC)$ the compensation cost and

$$h^*(\gamma, \mathbf{p}) = \frac{f^*(\gamma, \mathbf{p})}{1 - f^*(\gamma, \mathbf{p})} \quad (29)$$

the Laplace transform of the renewal intensity or failure rate with $f^*(\gamma, \mathbf{p}) = \int_0^\infty \exp[-\gamma t] f(t, \mathbf{p}) dt$ the Laplace transform of the density $f(t, \mathbf{p})$ of the times between failures. The compensation cost H_F will be discussed below.

If, in particular, there is a stationary stream of adverse Poissonian events (earthquakes, storms, fires, etc.) with occurrence rate λ and $P_f(\mathbf{p})$ the failure probability in an adverse event it is (see [71] and [62])

$$h^*(\gamma, \mathbf{p}) = \frac{\lambda P_f(\mathbf{p})}{\gamma} \quad (30)$$

so that $r(\mathbf{p}) = \lambda P_f(\mathbf{p})$ is the failure rate.

If the facility is given up after failure or a finite service time t_s we have [78]

$$Z(\mathbf{p}) = b \int_0^{t_s} \frac{1 - \exp[-\gamma t]}{\gamma} f_1(t, \mathbf{p}) dt - C(\mathbf{p}) - (H_M + H_F) \int_0^{t_s} \exp[-\gamma t] f_1(t, \mathbf{p}) dt \quad (31)$$

where $f_1(t, \mathbf{p})$ is the density of the time to first failure.

For aging structures (increasing risk function) with preventive maintenance by perfect repairs at regular intervals a (age replacement strategy) it is [78]

$$Z_M(\mathbf{p}, a) = \frac{b}{\gamma} - C(\mathbf{p}) - \frac{(C(\mathbf{p}) + H_M + H_F) f^{**}(\gamma, \mathbf{p}, a) + R(\mathbf{p}) \exp[-\gamma a] \bar{F}(\mathbf{p}, a)}{1 - (f^{**}(\gamma, \mathbf{p}, a) + \exp[-\gamma a] \bar{F}(\mathbf{p}, a))} \quad (32)$$

where $f^{**}(\gamma, \mathbf{p}, a) = \int_0^a \exp[-\gamma t] f(t, \mathbf{p}) dt$ is the incomplete Laplace transform of $f(t, \mathbf{p})$, $\bar{F}(\mathbf{p}, a)$ the probability of survival up to a and $R(\mathbf{p})$ the repair cost. $R(\mathbf{p})$ usually is much smaller than the failure cost $C(\mathbf{p}) + H_M + H_F$ [79]. Here, (perfect) repairs (renewals) are undertaken either after failure or at age a . Eqs. (28), (31) and (32) are only examples of a rich spectrum of objective functions based on the renewal model (see [78] and [79]). For example, multiple mode failures, non-constant benefit rates, block replacements, renewals following indicative inspections etc. can be handled. Non-constant discounting is discussed in [67].

In accordance with economic theory benefits and (expected) cost should be discounted by the same rate. While the owner or operator may take interest rates from the financial market the assessment of the interest rate for a cost-benefit optimization in the name of the public is difficult. The requirement that any objective function must be non-negative leads immediately to the conclusion that constant interest rates must have an upper bound γ_{\max} depending on the benefit rate

$b \approx \beta C(\mathbf{p})$ (see [28]). For example, a special objective derivable from eq. (28) is:

$$\frac{\beta C(\mathbf{p})}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F) \frac{\lambda(\mathbf{p})}{\gamma} \geq 0 \quad (33)$$

where $C(\mathbf{p})$ are (re-)construction cost, H the damage cost, $\lambda(\mathbf{p})$ the (Poissonian) failure rate and \mathbf{p} some parameter. Therefore, by solving the equality for γ and given (optimal) $\mathbf{p} = \mathbf{p}^*$

$$\gamma < \gamma_{\max} < \beta - \lambda(\mathbf{p}) \left(1 + \frac{H_M + H_F}{C(\mathbf{p})} \right) \quad (34)$$

implying $\gamma < \beta$ for $\lambda(\mathbf{p}) \ll \beta$. It follows that the benefit rate β must be slightly larger than γ_{\max} . The public benefit rate is closely related to the real per capita economic growth. From eq. (33) one also concludes that there must be $\gamma > 0$ because the limit $\gamma \rightarrow 0^+$ is $\pm\infty$ or at least undefined. Generalization to time-dependent benefit and/or interest rates is straightforward.

The quantities N_{PE} and k in the compensation cost are the number of potentially endangered people and the probability of being killed in a failure event, respectively. They can only be estimated reliably for a specific project. k must be estimated taking account of the cause of failure, the severity and suddenness of failure, the possibilities for escape, possibly availability and functionality of warning and rescue systems, etc. The probability k can vary between less than 1/10000 and 1. For example, estimates show $k = 0.01$ to 0.1 for earthquakes [17], $k = 10^{-6}$ to 0.1 for floods [36], $k = 0.1$ for building fires [24] and $k = 0.02$ or less to 0.7 for large fires in road tunnels [52]. A summary of the estimation methodology is given in [37]. N_{PE} is not the maximum possible number of people in a unit but the expected number in an event by taking into account their time-averaged presence. For example, for living houses and flats one would expect around 0.5 persons per room, for hospitals 1 to 2 beds per room but for retail premises 10 to 20 persons per 100 m² gross sales area. Neglect of compensation cost H_F in eq. (28), (31) and (32) is ethically indefensible.

The legal situation for criteria of the type (21) for the owner or operator of a facility is **absolute** (strict) liability. The case of liability (due to negligence and/or intention) is not of interest since liability and compensations for loss of life and health is a matter for the courts. For absolute liability owners or operators additionally have to consider that they themselves or their insurance can compensate the dependents of the victims in an accident (plus, possibly, compensation for non-pecuniary damage). This is essentially the value of the lost earnings or the so-called societal human capital [64]. It is computed from

$$SHC = g \int_0^{a_u} \ell(a) h(a, n) da \approx g \frac{\ell}{2} \quad (35)$$

where $\ell(a) = \ell_d(a, 0, 0)$ the remaining life expectancy at age a . It is useful to average over the age-distribution $h(a, n)$. Alternatively, one can integrate eq. (10) to obtain the societal compensation cost

$$SLSC = \int_0^{a_u} LSC(\ell(a)) h(a, n) da \approx LSC\left(\frac{\ell}{2}\right) \approx SHC \quad (36)$$

with

$$LSC(\ell_r) = g \left[1 - \left(1 + \frac{\ell_r}{\ell} \right)^{-\frac{1}{q}} \right] \ell_r \quad (37)$$

and where ℓ_r is the number of life years saved. It is important that the remaining life expectancy in an event is undiscounted in order to avoid double discounting. Depending on the legal and taxation

conditions *SHC* or *SLSC* may also be computed with the full GDP. Small variations in q are numerically insignificant.

7.2 Risk Acceptance Criteria

Society, however, has to protect life and health of an anonymous person against all involuntary risks. For technical facilities the change in life expectancy or mortality must be replaced by a change in the failure rate, i.e. [64]

$$dm = kdr(\mathbf{p}, t) \quad (38)$$

where k is the probability of being killed in or by the facility in an adverse event and $r(\mathbf{p}, t)$ the failure rate dependent on a parameter set \mathbf{p} .

From renewal theory it is known that the (unconditional) failure rate corresponding to the renewal intensity is (dependance on \mathbf{p} omitted for the moment)

$$r(t) = \sum_{i=1}^{\infty} f_i(t) \quad (39)$$

where $f_i(t)$ is the density function of the time to the i -th renewal (failure). As pointed out in [19] $r(t)$ has a limit $1/E[T]$ for $f(t) \rightarrow 0$ if $t \rightarrow \infty$ and $E[T]$ the mean time between failures. In approaching the limit it can be strictly increasing, strictly decreasing or oscillate in a damped manner around $1/E[T]$. Ordinary renewal processes then tend to be large around $t = E[T], 2E[T], \dots$ and small around $t = 0, \frac{3}{2}E[T], \frac{5}{3}E[T], \dots$. The oscillations die out more rapidly for larger dispersions of the failure distribution. In many examples oscillations have been found when the risk function is increasing. Unfortunately, eq. (39) has closed-form solutions only for very few special failure models (for example, for the exponential distribution function [19], the normal distribution function [26], the Erlang distribution function [57] and the Weibull distribution function [76]) which, at most, may be used as approximations for realistic failure models. A particularly suitable numerical method for the calculation of the renewal function of other, possibly only numerical distribution functions $F(t)$ for $H(t) = E[N(t)] = F(t) + \int_0^t H(t-u)dF(u)$ is proposed in [5] from which $r(t) = \frac{dH(t)}{dt}$ (see also [79]).

However, there are two simple approximate results. For quite general failure models there is an asymptotic result

$$r(\mathbf{p}) = \lim_{t \rightarrow \infty} r(\mathbf{p}, t) = \frac{1}{E[T]} \text{ if } \lim_{t \rightarrow \infty} f(t) = 0 \quad (40)$$

For aging components with increasing risk function the following bounds on the renewal function are given in [6] p. 54

$$\frac{t}{E[T]} - 1 \leq \frac{t}{\int_0^t (1 - F(\tau))d\tau} - 1 \leq H(t) \leq \frac{tF(t)}{\int_0^t (1 - F(\tau))d\tau} \leq \frac{t}{E[T]} \quad (41)$$

The upper bound in eq. (41) turns out to be remarkably close to the exact result for small t . Under suitable conditions one also has

$$\begin{aligned} r(t) &= \frac{d}{dt} H(t) \leq \frac{d}{dt} \frac{tF(t)}{\int_0^t (1 - F(\tau))d\tau} = \\ &= \frac{1}{\int_0^t (1 - F(\tau))d\tau} \left(F(t) + tf(t) - \frac{tF(t)(1 - F(t))}{\int_0^t (1 - F(\tau))d\tau} \right) \end{aligned} \quad (42)$$

Again, the upper bound for eq. (42) is found to be very close to the exact result up to approximately $E [T]$. It also approaches the limit $1/E [T]$ for large t . The lower bound obtainable from eq. (41) by differentiation is generally less useful.

For systematically renewed but not maintained facilities it is now proposed to use the asymptotic failure rate $r(\mathbf{p}, t) = r(\mathbf{p}, t \rightarrow \infty)$ in eq. (40) and eq. (38) motivated by the constancy of the failure rate for large t . For Poissonian failure processes it is exactly constant for all times. For most other failure processes of practical interest the failure rate rapidly approaches its constant asymptotic value starting from zero. For facilities given up after failure or after a finite service time one needs to compute $r(\mathbf{p}, t_s)$. Finally, for facilities with preventive maintenance at regular intervals as in eq. (32) one should compute $r(\mathbf{p}, a)$, either numerically or by the upper bound eq. (42). It may be debated whether one even should use some time average in the last two cases.

Making use of eq. (21) this results for a given project, N_{PE} persons potentially endangered and k the probability of being killed in the failure event in criteria of the type

$$\begin{aligned} dC_Y(\mathbf{p}) &= -dg \geq -\frac{g}{q} E_A \left[\frac{d\ell_d(A, \rho, \delta)}{\ell_d(A, \rho, \delta)} \right] N_{PE} \\ &\approx -\frac{g}{q} C_{\Delta\bar{\ell}} N_{PE} dm = -G_{\Delta\bar{\ell}} N_{PE} dm = -G_{\Delta\bar{\ell}} k N_{PE} dr(\mathbf{p}, t) \end{aligned} \quad (43)$$

where $dC_Y(\mathbf{p})$ are the incremental yearly cost for a risk reduction by $dr(\mathbf{p})$. This quantity may also be denoted by *societal willingness-to-pay* per year (*SWTP*). In particular, the quantity $G_{\Delta\bar{\ell}}$ may also be interpreted as the *societal value of a statistical life* (*SVSL*) (see eq. (23)). Age-averaged willingness-to-pay is the correct quantity to use as it must be assumed that a representative cross-section of the population is endangered by the event-type hazard ⁸. Eq. (43) has to be applied for each hazard and failure mode if there are more than one ⁹.

⁸ If applied to specific projects all cost must be raised at the decision point. Then, $dC_Y(p)$ may include the cost of financing (as a constant annuity) such that $dC_Y(p) = dC(p) \frac{\exp[\gamma]-1}{1-\exp[-\gamma t_f]}$ where t_f is the credit period (or payback period) and $dC(p)$ is building cost at $t = 0$. g in $G_x(\rho, \delta)$ grows in the long run approximately exponentially with rate δ , the rate of real economic growth in a country so that over the service time $[0, t_s]$ the total is $G(t_s) = \int_0^{t_s} g \exp[\delta t] dt = g \frac{\exp[\delta t_s]-1}{\delta}$. Credit period and service time may differ. Returning to yearly payments we have

$$\begin{aligned} dC(p) \frac{\exp[\gamma]-1}{1-\exp[-\gamma t_f]} &\geq \frac{\exp[\delta t_s]-1}{\delta} \frac{g}{q} C_{\Delta\bar{\ell}} k N_{PE} dr(p) \\ &\text{or} \\ dC(p) &\geq \frac{1-\exp[-\gamma t_f]}{\exp[\gamma]-1} \frac{\exp[\delta t_s]-1}{\delta} \frac{g}{q} C_{\Delta\bar{\ell}} k N_{PE} dr(p) \end{aligned}$$

The acceptability criterion would now depend on project specific parameters like t_f, t_s and γ . For example, for $t_f = t_s = 25, \gamma = \delta = 0.02$ the factor on the right hand side accounting for the additional financing cost and economic growth is 1.7 from a factor of 1.3 is contributed by economic growth. In this case financing cost and increase in g due to economic growth are about the same so that the effects on the acceptability criterion cancel approximately.

⁹ The concept carries over to systems. If the system is reduced to a series system the failure rate is computed from eq. (39) where $f_i(t) = \frac{dF_i(t)}{dt}$ with

$$F_i(t) = P \left(\bigcup_{j=1}^n \{T_{i,j} \leq t\} \right) = 1 - P \left(\bigcap_{j=1}^n \{T_{i,j} > t\} \right) = \sum_{j=1}^n P \left(\{T_{i,j} \leq t\} \cap \bigcap_{k \neq j} \{T_{i,k} > t\} \right) \leq \sum_{j=1}^n P(T_{i,j} \leq t)$$

for n modes in series. The second last formula shows that the renewal function and so the renewal intensity is, in fact, the sum of the corresponding quantities of the individual modes. In many cases, however, one must use numerical techniques as given in [5]. Computational techniques can be found in [78] [79]. For independent failure modes one has the simple result $F_i(t) = \sum_{j=1}^n F_{i,j}(t) \prod_{k \neq j} (1 - F_{i,k}(t))$

It is important to note that the additional risk reducing investment $dC_Y(\mathbf{p})$ depends on the change $dr(\mathbf{p})$ of the failure rate and not on its absolute value. If $C_Y(p)$ and $r(p)$ are scalar and differentiable one also has $dC_Y(p) = \frac{dC_Y(p)}{dp} dp$ and $dr(p, t) = \frac{dr(p, t)}{dp} dp$. Division of both sides of eq. (43) by dp tells us how it can be solved in applications, i.e. by $\frac{dC_Y(p)}{dp} \geq -G_{\Delta\bar{\ell}} k N_{PE} \frac{dr(p, t)}{dp}$. For a vectorial parameter \mathbf{p} one best solves the criterion as an optimization task

$$\text{Minimize: } S(\mathbf{p}, t) = C(\mathbf{p}) + G_x k N_{PE} r(\mathbf{p}, t) \quad (44)$$

recognizing that eq. (43) is the first-order optimality condition of eq. (44) if no cost-benefit optimization is performed. Cost-benefit optimal facilities may or may not fulfill criterion (43)¹⁰.

8. Risk Reduction for Toxic Substances

For continuously effective, ubiquitous, environmental pollutants in air, soil or water dm or $\frac{d\ell_d(A, \rho, \delta)}{\ell_d(A, \rho, \delta)}$ usually are related to the dose of those exposed and risk reduction measures work on limiting or reducing the dose [38]. Frequently, the dose accumulates over time increasing mortality accordingly. Also, latency has to be considered appropriately and there may be a delay time until an (monetary) intervention shows effect [34]. Additional considerations about the emitting source, the transport mechanisms and the exposure to the hazard have to be performed. Let, as a simple example,

$$dm(a) = f(x) = \kappa x \begin{cases} a - T & \text{for } a > T \\ 0 & \text{for } a \leq T \end{cases} \quad (45)$$

be the dose-mortality relationship with x the dose and κ the so-called slope factor, $(a - T)$ takes account of the cumulative nature of exposition to a time-invariant dosis and T is a deterministic latency period. A deterministic latency period clearly is an approximation but data are almost completely missing for a better model. Furthermore, D is the (deterministic) delay time between payments for risk reduction and the time where they become effective. Eq. (21) is a convenient starting point. In fact, the only difference is a slightly changed relation of the type eq. (24). For example, eq. (24) changes for a cumulative dose according to eq. (45) and additive mortality changes, i.e. $\mu_\Sigma(a) = \mu(a) + dm(a)$, into

$$C_{\Delta\bar{\ell}}(\rho, \delta, D, T) = \int_0^{a_u - D} \frac{\ell_d(\rho, \delta, a, T, D, y) - \ell_d(\rho, \delta, a, T, D, 0)}{\ell_d(\rho, \delta, a)} h(a, n) da \quad (46)$$

with

$$\ell_d(\rho, \delta, a, T, D, y) = \int_{a+D}^{a_u} \exp \left[- \left(\int_a^t \rho(\theta) d\theta + \delta(t - a) + \int_{a+D}^t \kappa y \begin{cases} \theta - T & \text{for } \theta > T \\ 0 & \text{for } \theta \leq T \end{cases} d\theta \right) \right] dt$$

if it is assumed that payments for risk reduction start at the decision point. If payments start after the delay time the integral over the discount rate in eq. (46) starts at $a + D$. But there is a large spectrum of conditions under which exposition to toxic substances occurs, accumulates and causes fatalities. And there are different kinds of mortality reduction schemes in use including proportional mortality reductions.

¹⁰ For example, for eq. (28) with (30) the first-order optimality condition is

$$\frac{dZ(p)}{dp} = -\frac{dC(p)}{dp} \left(1 + \frac{\lambda P_f(p)}{\gamma} \right) - (C(p) + H_M + H_F) \frac{\lambda}{\gamma} \frac{dP_f(p)}{dp}$$

For $\lambda P_f(p) \ll \gamma \ll 1$ there is $p_{opt} > p_{lim}$ or $\lambda P_f(p_{opt}) < \lambda P_f(p_{lim})$ if $\frac{C(p) + H_M + H_F}{\gamma} \geq G_{\Delta\bar{\ell}} k N_{PE}$. This is frequently the case.

The demographic constants $C_{\Delta\bar{\ell}}(\rho, \delta, D, T)$ and $C_{\pi\bar{\ell}}(\rho, \delta, D, T)$ as well as the constants $C_{\Delta}(D)$ and $C_{\pi}(D)$ decay with increasing delay period as shown in fig. 8 for an example with $dm(a) = \Delta$ or $dm(a) = \mu(a)\pi$ and $T = 0$. In the left figure discounting is from age a . The right hand figure shows the influence of the starting point of discounting. Fig.9 shows the effect of a latency period and cumulative dose for the relationship in eq (45) with $\kappa = 3 \cdot 10^{-5}$. In both figures age-averaging is taken into account.

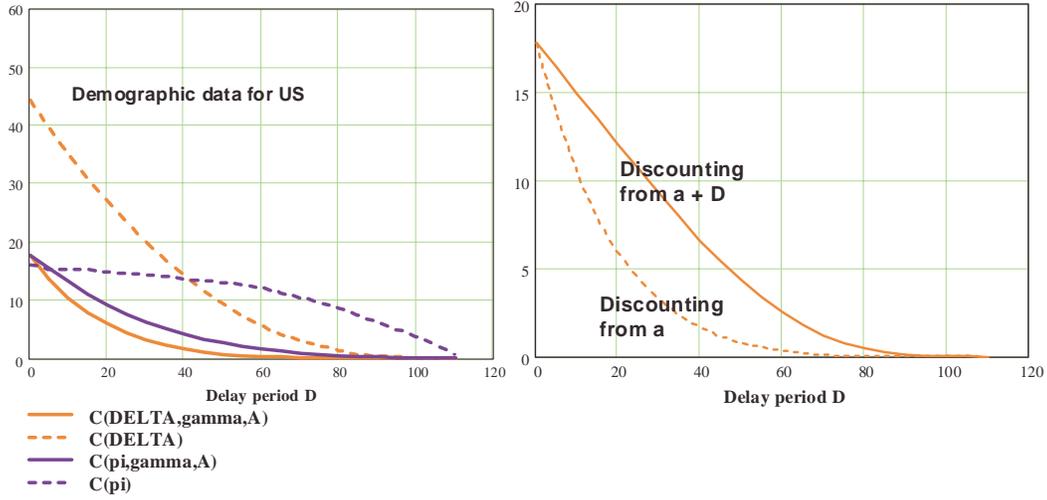


Figure 8: Demographic constants $C_{\pi\bar{\ell}}(\rho, \delta, D, T)$ and $C_{\Delta\bar{\ell}}(\rho, \delta, D, T)$ over delay period (latency period $T = 0$)

Here, we see the combined effect of discounting and a smaller proportion of the affected population. Therefore, for a yearly dose reduction scheme x

$$dC_Y = -dg \geq -\frac{g}{q} E_A \left[\frac{d\ell_d(A, \rho, \delta)}{\ell_d(A, \rho, \delta)} \right] \approx -\frac{g}{q} C_{x\bar{\ell}}(\rho, \delta, D, T)x \quad (47)$$

9. Predictive Cohort Life Tables

Setting risk acceptability criteria also has a sustainability aspect because the next or even the far future is concerned. Period life tables have been used in [63] and [66]. So-called cohort life tables certainly would be better as they reflect the common trend towards larger life expectancies and more compact age distributions. Since we are interested in future risks we have to extrapolate into the future. Time- and age-dependent mortalities can be obtained by extrapolating from a sequence

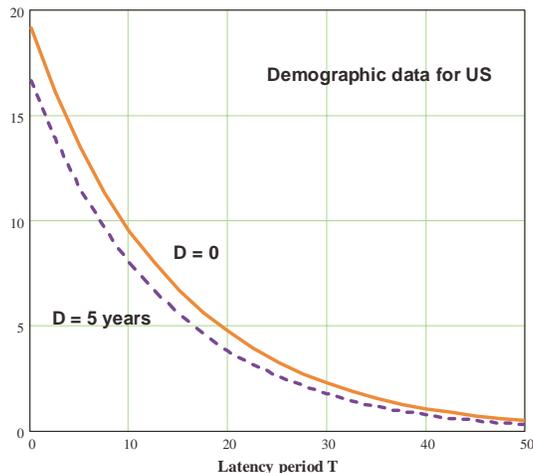


Figure 9: Demographic constant $C_{\Sigma\bar{\ell}}(\rho, \delta, D, T)$ for cumulative dose over latency period

of historical period life tables so that

$$\mu_{\vartheta,y}(a) = \mu_y(a)b(a)^{\vartheta+a-y} \quad (48)$$

where y is the reference year, i.e. the last year for which a period table is available and $\vartheta \leq y$ is the year of birth. Unfortunately, cohort life tables do exist only for a few countries. Cohort life tables, for example, yield about 7% larger life expectancies at present. Predictive cohort table must be constructed, for example for $y = \vartheta = 2000$ and $0 \leq a \leq 110$. Results are collected in table 7 for thirteen developed countries. An uninterrupted sequence of period life tables must be available for at least the last 50 years but not longer than 100 years so that extrapolations for the age dependent mortalities can be performed. Although even longer sequences of period tables are available, only tables for the last 100 years are used in order to take account of the reduced infant mortality. The data used are all from [13] where some more countries are included. Clearly, such extrapolations are based on the assumption that the observed demographic trends continue throughout the next 100 years. Trends in all other demographic parameters are not taken into account but must be expected. Their effect, however, is small. For example, if population growth n is set to zero resulting in a small change in the age structure of a population no noticeable change in $G_{\Delta\bar{\ell}}$ is observed.

10. Data

The quality of the data varies. Data for the GDP are rather consistent when using inflation-free data from national statistical offices, the World Bank, the OECD or the United Nations. Somewhat less reliable and sometimes contradictory data are available for the parts of the GDP used for private and government consumption. Below, the data in [82] are used showing a lower value g_l being private consumption alone and an upper value g_u as the sum of private and government consumption. Similar data are collected in [4]. Presumably, the amount available for risk reduction is somewhere in between. Historical data on economic growth rates have been found in [46]. They have been averaged over the years 1870 to 1992. The demographic data (life tables, life expectancies, population growth rates) are excellent, especially when taken from [13]. Abbreviated period life tables for 191 countries are available in [42]. The data on working times, however, are

not very precise. Nathwani et al. [44] estimated $w = 1/8$ which includes 1 hour travel time per working day and assumes a life working time of 45 years. Detailed labor statistics can be found in [51] and [22]. But Evans et al. [23], for example, and other specialists from the labor statistics field continue to underline that almost all published data are not suitable for cross-country comparisons due to different evaluation methods in the different countries. Among the more important factors are: is part time work included?, is the household survey technique or the establishment survey technique applied?, is the work time of the self-employed considered?, is unemployment considered?, are multiple job hours included?, is overtime work included?

In fig. 10 the quantity w as estimated in [44] is plotted for some industrialized countries. They are based on various official statistics [51]. In this figure one hour travel time per working day and a life working time of 45 years has been assumed as in [44]. Statistics of the life working time (i.e. the total hours of work in a life) appear to be almost inexistent. It is believed that the life working time is actually smaller in many countries as suggested in table 5 for an example. The values in this table look reasonable but the data source is unclear.

Durations in years	1906	1950	1990
Childhood & education	14.2	14.2	19.4
Life working time	43.8	45.8	40.6
Retirement	4.8	11.4	16.4
Sum = life expectancy	62.8	71.4	76.4

Table 5: Historical life working times for Germany [27]

The quantity w clusters closely around a global mean confirming the estimate in [44]. However, it can be observed quite generally that societies with larger GDP generally work less whereas people in countries with smaller GDP work more in order to increase utility of consumption - especially if one considers also less developed countries [63]. But there are exceptions from the general trend. In some countries with large GDP preference is given to large earnings and thus large consumption whereas other societies prefer larger leisure time versus somewhat less consumption. Obviously, other secondary factors also affect the value w . Because the value of w turns out to be a rather important parameter it is worth discussing its value in more detail.

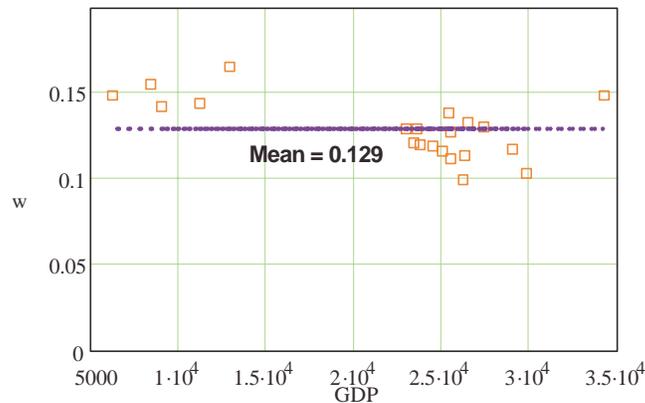


Figure 10: Life working time for various countries according to [51] assuming a life working time of 45 years and one hour of travel time per working day.

A promising alternative is as follows. While the travel time assumption appears to be generally acceptable we are proposing to determine w from

$$w = \frac{\text{labor force in economy}}{\text{population}} * \frac{\text{yearly worked hours}}{24 \cdot 365} * \frac{9}{8} \quad (49)$$

using the data in [51] where the yearly worked hours are determined by the total number of yearly worked hours in an economy divided by the number of people in employment. In the formula a normal work day of 8 hours plus travel time of one hour is assumed (factor 9/8). In this way of determining w several serious data problems are avoided. The number of wage earners (employed and self-employed) excludes the unemployed. Assuming that all unemployed would work if there is work and there is a functioning system for fair unemployment compensation the labor force is the correct numerator in this formula¹¹. However, there are also some good reasons to exclude the unemployed from the labor force. The new data are shown in figure 11. They are by almost by one third smaller than in fig. 10.

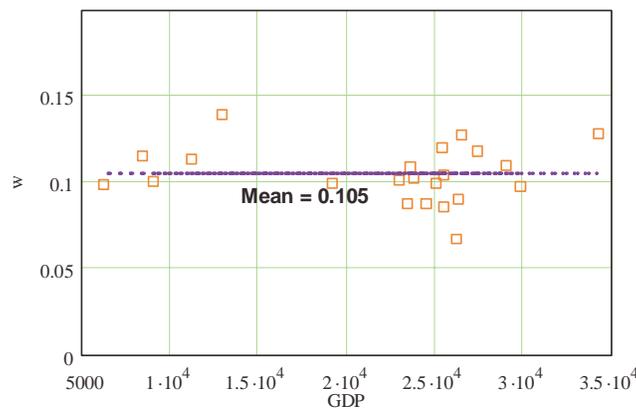


Figure 11: Life working time for various countries according to [51] related to labor force and including one hour of travel time per working day.

11. Social Indicators for a Number of Countries

Before a number of social indicators are given we investigate the effect of discounting and age-averaging as well as using predictive cohort tables as opposed to recent period tables in more detail. Table 6 compares for constant ρ the results for period and predictive cohort tables for proportional and constant mortality reduction. For information, the coefficients without any discounting and age-averaging, for age-averaging only and for discounting only are also given. Discounting is

¹¹ It is interesting to use the concept eq. (49) to compute the life working time in years from

$$\frac{\text{labor force}}{\text{population}} \text{ywh per employee} = \frac{\text{life working time}}{\text{life expectancy}} \text{ywh per employee}$$

For example, for the USA one calculates 39 years, for Germany 38.6 years, for Japan 43.2 years but for the Netherlands only 30.8 years based on data in [51]. However, the data for the Netherlands are for dependent employment only and do not include overtime work.

performed according to eq. (14) with $\rho = 0$.

	France	Germany	Japan	USA
δ in %	1.9	1.9	2.7	1.8
ρ in %	1	1	1	1
From period tables	$C_x, C_{xd}, C_A, C_{x\bar{l}}$			
Proportional mortality change	17,6,32,21	14,5,29,19	16,4,34,21	20,7,34,22
Constant mortality change	40,25,24,16	40,25,23,16	41,22,24,15	39,25,25,17
From predictive cohort tables	$C_x, C_{xd}, C_A, C_{x\bar{l}}$			
Proportional mortality change	13,4,25,18	12,4,25,16	11,2,26,16	16,6,28,18
Constant mortality change	44,27,25,17	44,27,26,18	47,25,26,16	44,28,28,19

Table 6: Comparison of demographic constants from period and predictive cohort tables

(C_x : no discounting and no age-averaging, C_{xd} : discounting only, C_A : age-averaging only, $C_{x\bar{l}}$: discounting and age-averaging)

It is seen that the differences in the coefficients for period and predictive life tables are relatively small, at least for the countries considered. Discounting only reduces the coefficients significantly whereas age-averaging increases them for proportional mortality reductions but reduces them for constant mortality reductions. Here again, one observes considerable differences between the two mortality reduction schemes. However, applying both age-averaging and discounting levels out the differences.

Numerical values for various economic and demographic quantities are given in table 7. Several countries have been chosen in order to see the effect of the different factors. Only those countries have been chosen which are economically and demographically comparable and for which the data required are complete. The data for GDP, g_l , g_u , δ and n are discussed in detail in [64] and above. The growth rate δ is an average between 1870 and 1992. It certainly would be inappropriate to take only the years between 1950 and 1992 where growth was almost doubled. Eq. (7) with $\beta = 0.7$ is used throughout. It certainly varies a little from country to country but suitable data are very difficult to get for many countries and small variations are insignificant. Monetary quantities are given in PPPUS\$ (corrected for purchasing power parity). Some minor inconsistencies in the data could not be removed, however. This concerns primarily the worktime estimates¹². But it must be emphasized that variations of up to $\pm 30\%$ in the value of a statistical life hardly make a difference in applications.

The computed values for $SLSC$ must be considered as lower bounds. If working time is made dependent on t , i.e. by a suitable function fitting the trends in [46] [50] and [51] sufficiently well, one finds that $G_{\Delta\bar{l}}$ goes down by 0.1 to $0.3 \cdot 10^6$. But remember that the future development of $w(t)$ depends on many factors which are very difficult to predict (see table 3 and 4 and the related comments). Strictly speaking, a time-dependent utility function $u(c, t)$ is no more admissible in our simplified reasoning. A more general and more complicated theory would be needed involving dynamic optimization (see, however, the discussions for fig. 4 and footnote 4). Therefore, it is assumed that w is already at a stationary value. In table 7 one hour travel time per working day is added to the observed values as computed by eq. (49). For all considered countries $\rho = 0.03$ is assumed in eq. (12). The effect of reasonable variations of ρ is small. The results for C_{Δ} , C_{π} , $C_{\Delta\bar{l}}$ and $C_{\pi\bar{l}}$ show remarkable consistency among the countries considered. Multiplying those constants by g/q gives the constant in the acceptability criterion for various mortality reduction schemes. As an example, the $G_{\Delta\bar{l}}$ -values are given for $(g_l + g_u)/2$ as a compromise and q in eq. (7) with w by

¹² In particular, the work time estimate for the Netherlands in [51] appears unrealistically low. Also, the data for the USA are probably too high as compared to the other countries.

eq. (49). If $\rho = 0$ but discounting with δ is maintained $G_{\Delta\bar{\ell}}$ increases by 10 to 20%.

Country	GDP ¹⁾ , g_l ²⁾ , g_u ³⁾	δ ⁴⁾	n ⁵⁾	e ⁶⁾ , e ⁷⁾	w ⁸⁾	LQI ⁹⁾	SLSC ¹⁰⁾	$C_{\Delta}, C_{\pi}, C_{\Delta\bar{\ell}}, C_{\pi\bar{\ell}}$	$G_{\Delta\bar{\ell}}^{11)}$
Canada	27330,16040,21560	2.0	0.99	78,84	0.12	1132	$7.6 \cdot 10^5$	43,18,17,21	1.8
USA	34260,22030,27240	1.8	0.90	77,86	0.13	1141	$1.0 \cdot 10^6$	44,16,18,18	2.1
Austria	26330,14800,19800	1.8	0.24	78,85	0.10	1241	$6.5 \cdot 10^5$	43,12,16,17	1.9
Belgium	27470,17340,21340	1.5	0.16	78,85	0.08	1329	$7.7 \cdot 10^5$	43,12,16,18	2.4
Czech Rep.	12900, 6730, 9225	1.4	-0.07	73,77	0.14	909	$2.5 \cdot 10^5$	40,14,16,21	0.54
Denmark	27250,13800,20700	1.8	0.3	77,82	0.09	1196	$5.6 \cdot 10^5$	42,13,16,17	1.7
Finland	24570,13000,18130	2.1	0.16	77,83	0.11	1164	$5.4 \cdot 10^5$	42,13,15,18	1.3
France	24470,14660,19380	1.9	0.37	78,84	0.09	1195	$6.7 \cdot 10^5$	43,13,16,18	1.9
Germany	25010,14460,19310	1.9	0.27	78,86	0.10	1136	$6.5 \cdot 10^5$	44,12,16,16	1.9
Italy	23400,14460,18280	2.0	0.07	79,82	0.09	1197	$5.9 \cdot 10^5$	42,12,15,17	1.8
Netherlands	26170,15470,19050	1.5	0.55	78,84	0.07	1347	$7.1 \cdot 10^5$	43,16,17,20	2.8
Norway	29630,14070,20060	2.1	0.49	79,82	0.10	1239	$6.2 \cdot 10^5$	42,12,16,17	1.8
Spain	19180,11890,15000	1.8	0.10	78,85	0.10	1106	$5.1 \cdot 10^5$	43,10,16,17	1.3
Sweden	23770,12620,18750	1.9	0.02	79,84	0.10	1142	$5.1 \cdot 10^5$	42,11,15,17	1.5
Switzerland	29000,17700,21840	1.9	0.27	79,85	0.11	1158	$7.7 \cdot 10^5$	43,13,16,19	1.8
UK	23500,15140,19920	1.3	0.23	78,82	0.11	1102	$6.0 \cdot 10^5$	42,12,18,17	1.7
Japan	26460,15960,18520	2.7	0.17	80,92	0.13	1127	$7.2 \cdot 10^5$	47,10,15,16	1.3
N. Zealand	18530,11750,14560	1.2	1.14	79,84	0.12	1123	$5.7 \cdot 10^5$	43,15,19,18	1.3

Table 7: Social indicators for some countries: ¹⁾ in PPPUS\$ in 1999 [91], ²⁾ private consumption in PPPUS\$ [82], ³⁾ private plus government consumption in PPPUS\$ [82], ⁴⁾ average yearly economic growth per capita in % for 1870-1992 after [46], ⁵⁾ population growth (2000) in % [16], ⁶⁾ life expectancy in 2000, ⁷⁾ life expectancy for those born in 2000, ⁸⁾ estimates based on [51] including 1 hour travel time per working day, ⁹⁾ eq. (8) with $g=g_l$, data from 2000, ¹⁰⁾ SLSC computed with g_l and age-averaged life expectancies according to eq. (36) for predictive age distributions, ¹¹⁾ computed with $(g_l+g_u)/2$ from predictive cohort life tables, discount rate according to eq. (12) with $\rho = 0.03$ and $\epsilon = 1 - q$, Δ indicates constant additive mortality changes at all ages, in 10^6 PPPUS\$.

12. Empirical Verification of Constant in Acceptability Criterion

There have been many attempts to estimate this quantity indirectly (among the rich literature for this subject see, for example, [86], [87], [88], [80], [48] and [43]), mostly by estimating the cost of some life saving operation like limiting highway speed [3], installing smoke detectors in homes, using seat belts in cars or purchasing bicycle helmets. Also, the compensation in risky jobs by higher wages has been used [88] as well as surveys with respect to hypothetical risky situations, so-called contingent valuation studies [59] [32]. Also, revealed preference methods have been used. The values reported are between less than 1 Mill. US\$ and more than 10 Mill. US\$, i.e. more than 2 to 20 times as much as the (undiscounted) value of average lost earnings in case of a fatal accident at mid life. Tengs et al. [80] find cost of less than 1000 US\$ up to 100 Mill. US\$ for a large number of life saving interventions. Health related investments cluster around 4 to 6 Mill. US\$ [87]. Studies in the US tend to give somewhat larger values than European studies but not without exceptions. Contingent valuation studies tend to give about the same or slightly larger mean values but larger spread. Market studies (buying bicycle helmets, cigarette smoking) gave the smallest value. When comparing numbers one has to keep in mind that they have to be adjusted to year 2000. For example, values for year 1995 should be multiplied by 1.1 and values

for year 1985 by 1.6. The studies [89] , [49] , [48] and [11] among several other similar studies are so-called meta-analyses, i.e. analyses across several other studies. They should be especially valuable for our purpose. The studies in [89] , [49] , and [11] contain 60, 33 and 25, respectively, other studies. In fig. 12 and 13 some results collected in [89] and [49] are presented graphically showing means between 6 and 8 Mill. US\$ and the large scatter of the estimates. The distributions of VSL are highly skewed so that the corresponding medians are some 30% smaller. The estimates in [49] based on wage differentials in the USA and recomputed in [89] are significantly smaller in contrast to the findings in [90] . Their own estimates by Mrozek/Taylor [49] found a range between 1.5 and 2.5 Mill. US\$. The correlation coefficient as a crude measure of the dependence of VSL (or VOSL) on income (or GDP) is also given in the figures.

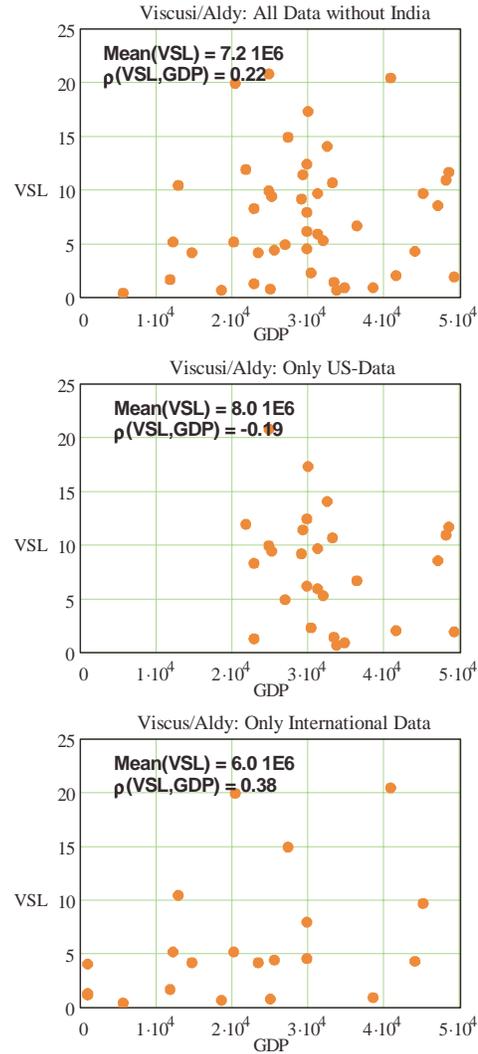


Figure 12: VSL (in year 2000 US\$) according to [89]

Viscusi/Aldy [89] and others found that the VSL generally increases with income (GDP) and decreases with age in accordance with our theory. Labor market studies yield slightly larger VSLs than housing and product studies. Some road safety investigations present relatively low values.

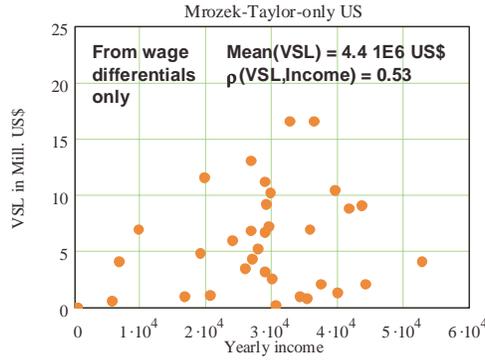


Figure 13: VSL according to [49]

Higher background mortality (smokers or other) decreases the VSL as well as higher risk levels which is not in accordance with the theory. Latency can decrease the WTP's considerably. WTP's for private goods (for example, safe cars) appear to be larger as compared with public goods. Viscusi/Aldy [89] discuss in depth why estimates depend on quite a number of factors and are different for compensating wage differentials, public health interventions and product markets. De Baeij et al. [11] also find large differences even in the narrow field of road safety (see Fig. 14). The variability in [89] or [11] may in large part be explained by differences in the methodologies used to estimate the value of life and which still are intensely debated in the relevant literature. But apart from methodological differences one has to recognize that individual and public risk perception, the leading factor in many studies, and the corresponding WTP's for the reduction of risks in the different areas must simply be considered as highly subjective. In addition, and this appears very obvious, many estimates are based on in part strongly biased samples (higher job risks are generally taken by healthy, male and young adults).

Some uncertainty exists about the precise meaning of the reported values of VSL . It appears as if the definition of VSL is not everywhere the same but the majority of studies starts from the definition in eq. (25) with $a = 0$, mostly in the form $VSL = WTP/dm$. Furthermore, it is rarely clear whether the empirical estimates take account of the age-dependency, consider age-averaging and/or a particular mortality reduction scheme and the level of risk reduction. It is also unclear whether discounting is at all considered and, if, at which rate. Nevertheless, if we simply take $VSL = \frac{g}{q}\ell$, the definition in eq. (26) without age-averaging and discounting for $a = 0$, there is excellent agreement in the means between theoretical and empirical estimates. But this observation may be just coincidental. The non-compatibility with some empirical estimates does not make our theory obsolete in view of the above remarks about biases in the data, methodological differences and the question of discounting.

Even if one considers the variations in g and w for the different countries it is clear that table 7 gives much smaller spread of the constants $G_{\Delta} = \frac{g}{q}C_{\Delta}$ or $G_{\Delta\bar{\ell}}$ or similar than the empirical estimates. Comparing our theoretical results with the empirical estimates shows reasonable agreement in the mean of the value of G_{Δ} , especially if computed with the full GDP, although G_{Δ} still falls into the lower half of the empirical estimates. Remember the constant C_{Δ} is about 2.5-times larger than $C_{\Delta\bar{\ell}}$. Age-averaging and discounting are responsible for the lower $G_{\Delta\bar{\ell}}$ or similar.

However, Viscusi/Aldy [89] and others made a disturbing observation. They determined the income elasticity of the VSL to be in the order of 0.5 with variations between 0.15 and something

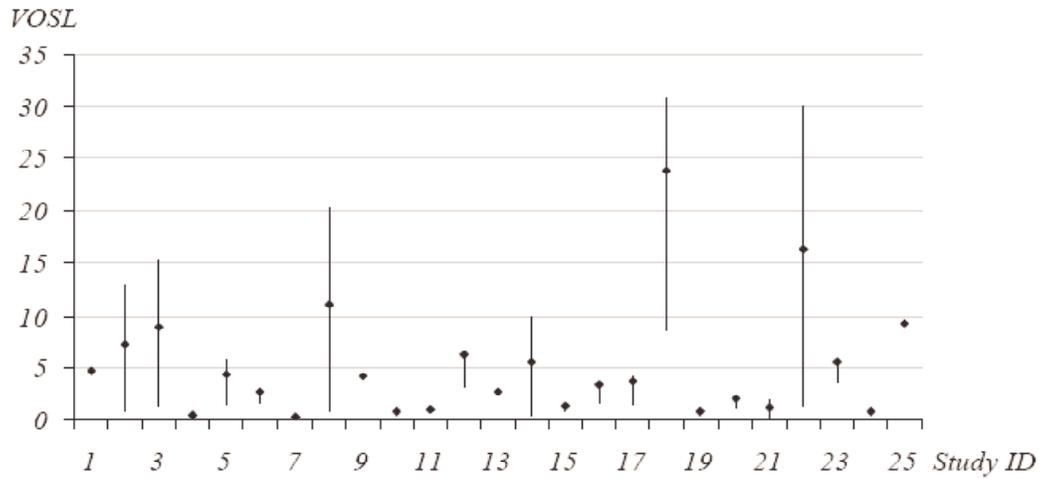


Figure 14: Mean, highest and lowest estimate of the VOSL (in Mill. 1996 US\$) in 25 road safety studies[11]

a little less than unity. Our theory predicts a value of one¹³. The observation in Viscusi/Aldy simply says that in many areas (for example, different labor markets) and by various methods (wage differentials or contingent valuation studies) people are more risk-prone than predicted by our theory. In another study, Cost/Kahn [18] recently determined statistically an income elasticity of 1.5 to 1.7 for historical wage differentials. Apparently, some inconsistencies are present in the empirical estimates. The reasons most likely are again systematically biased samples.

Viscusi/Aldy [89] also report about the value of a statistical injury. The studies are mostly from the labor market area in the US and Canada. The mean value is about 50000 US\$ per injury. Leaving out the extremes the values of the majority of studies concentrate in an interval of 20000 to 70000 US\$. This is significantly less than even the most modest estimates of *VSL*.

13. Conclusions

This study attempts to summarize recent findings how to set risk acceptability criteria in terms of life expectancy and economic parameters. Moreover, it provides a theoretical foundation for rationalizing the apparent huge diversity in empirical estimates of values of life savings, based on the willingness to pay approach. Many partial aspects of the theory are verified by empirical evidence so that the theory can be considered as sufficiently complete. So far the empirical findings in the last section neither support fully our theory nor can it be dismissed on the same grounds. What can be said is that the theory is unaffected by subjective factors, is rational, affordable and in agreement with ethical standards. Extreme catastrophic events are excluded if they are capable of changing significantly the economy or the structure of the population in a region. Such events are thinkable, examples being meteorite impact, earthquakes with magnitude beyond 8.5 in highly

¹³ Income elasticity of a quantity is defined as the relative change in that quantity divided by the relative change in income. Therefore,

$$\text{income elasticity} = \frac{\frac{dSVSL}{SVSL}}{\frac{dg}{g}} = \frac{dSVSL}{dg} \frac{g}{SVSL} = 1$$

populated areas or the maximum credible accident in a chemical or nuclear plant. They require additional considerations.

- There are two independently developed approaches for the quantification of the (societal) willingness-to-pay for risk reduction that is the health-related economical approach as advocated and outlined, for example, by Shepard/Zeckhauser [75] and the approach based on the societal life quality index (*SLQI*) as advocated by Nathwani et al. [44] and Pandey/Nathwani [55]. Although slightly different in the derivations they complement each other and lead to the essentially same numerical results under otherwise identical parameter assumptions and mortality reduction schemes.
- The key quantity is the societal willingness-to-pay for risk reduction. It is given as

$$SWTP = \frac{g}{q} E_A \left[\frac{d\ell_d(A, \rho, \delta)}{\ell_d(A, \rho, \delta)} \right] \approx \frac{g}{q} C_{x\bar{\ell}} dm$$

It is proportional to the monetary amount g available for risk reduction and the size of the risk reduction dm (as a mortality reduction) and inversely proportional to a risk aversion parameter q expressing an optimized ratio of labor and leisure time. The proportionality constant $C_{x\bar{\ell}}$ depends on economic and demographic factors but especially on the specific mortality reduction scheme x of a life saving intervention. The quantity $\frac{g}{q} C_{x\bar{\ell}}$ may be interpreted as societal value of a statistical life.

- One important parameter to be used is the part of the GDP available for public risk reduction. The GDP is considered as the presently best measure of welfare and wealth in an economical sense. Clearly, the part used for investments in order to renew deteriorating production means is not available if intra- and intergenerational equity is required. Whether the part of the GDP usually used by the public (state and communities) for internal and external safety, education, protection of national heritage and similar public duties can also be used can be debated. However, this may be interpreted as a component of life quality not directly covered by either of the approaches. Then, it must also be deducted from the GDP. The minimum available for risk reduction is the part of the GDP for private use. It is also debatable whether the amount available for risk reduction is only the excess of some minimum subsistence level of consumption because a level below is the same as being dead.
- Another important parameter is survival probability or life expectancy. If only event-type hazards are considered at the decision point at least all living at the decision point should be considered. This implies that predictive cohort life tables should be used. The time-horizon to be considered determines the prediction time. As a first approximation a time of around 100 years is proposed as prediction time. If event-type hazards to technical objects are considered remaining life expectancies should be considered and averaging needs to be performed over the affected age-groups in order to fulfill the intragenerational equity requirement. Toxic risks might require slightly different treatment.
- The third critical parameter is the effective life working time of the generations living up to the time-horizon. It is proposed to start from the present yearly working hours in official sources, for example in OECD sources. Travel times per working day between home and work may be added or not. In applications the fact that only a part, although the larger one, i.e about 70%, of the national production is due to labor should be taken into account.
- Discounting of future consumption is necessary for a growing (decreasing) population and a growing (decreasing) economy as well as for the other reasons of discounting. Exponential discounting should be performed by real time-averaged rates. This discounting should also ful-

fill sustainability requirements (intra- and intergenerational equity). Thereby, it may be assumed that future generations have the same preferences as the present generation. The minimum (real) discount rate is equal to the time-averaged economic growth rate per capita.

- The specific mortality reduction scheme for man-made or natural hazards is an important parameter. However, only schemes with constant mortality reductions at all ages can be considered as intragenerationally equitable for man-made and natural hazards.
- There is no explicit discounting in acceptability criteria involving yearly payments for risk reduction.
- If the LQI is used as an alternative for the HDI (ranking of countries) the form eq. (8) can be recommended with $g = GDP$ (all parameters at their present values).

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